CHAPTER 12

Energetics of Nuclear Reactions

Contents

Reactions between an atomic nucleus and another particle are called nuclear reactions. In some such reactions, new nuclei are formed (*nuclear transmutations*); in others the original nucleus is excited to a higher energy state (*inelastic scattering*); in a third class, the nucleus is unchanged (*elastic scattering*). Spontaneous nuclear transformations, which are involved in the radioactive decay of unstable nuclei, have been discussed in Chapter 4. In this chapter the emphasis is on the mass and energy relationships when a projectile interacts with a nucleus.

12.1. Conservation laws in nuclear reactions

We have discussed a number of laws governing nuclear processes. A summary of these laws provides us with a picture of the mechanics of nuclear interactions occurring below ~ 100 MeV/A, in which energy range the mass number is conserved.

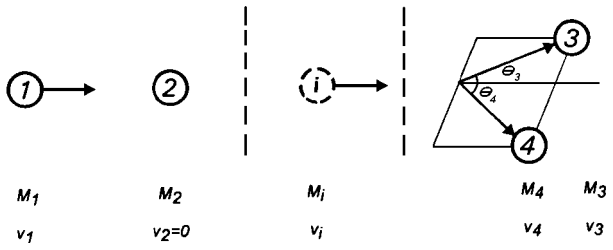

FIG. 12.1. Schematic picture of a nuclear reaction.

Let us begin by considering the simple process shown in Figure 12.1, in which a *projectile*, indicated by subscript 1, collides with a *target* atom, subscript 2, forming an intermediate system, subscript i. The intermediate system splits into the products, which are designated by subscripts 3 and 4. We call this a *nuclear reaction*. The velocity of the projectile v_1 is greater than zero, but the velocity of the target atom v_2 is made to be zero by using the target nucleus as the origin of the system of reference.

All the *conservation laws* derived in §4.2 are applicable to nuclear reactions. These are:

(a) the conservation of total energy:	$\Delta E = 0$	(4.2)
(b) the conservation of linear momentum:	$\Delta p = 0$	(4.4)
(c) the conservation of total charge:	$\Delta Z = 0$	(4.6)
(d) the conservation of mass number:	$\Delta A = 0$	(4.7)
(e) the conservation of spin:	$\Delta I = 0$	(4.9)

"Conservation of total energy" means that the total energy of the products must equal the total energy of the reactants, i.e. $E_{\text{products}} - E_{\text{reactants}} = \Delta E = 0$. For (b) it should be remembered that linear momentum is a vector property; thus

$$p_1 + 0 = p_3 \cos \theta_3 + p_4 \cos \theta_4 \tag{12.1a}$$

$$p_3 \sin \theta_3 = p_4 \sin \theta_4 \tag{12.1b}$$

Some of these conservation laws (e.g. (d) and (e)) are not always obeyed in high energy reactions in which new elementary particles may be formed. The kinetic energy equation $E_{\text{kin}} = \frac{1}{2} mv^2$ (2.5) was deduced by Newton in 1687 assuming that the mass of the particle was independent of its velocity. We have seen that this is not true at high particle velocities (cf. Fig. 4.2). For example the relativistic mass increase of a bombarding proton is about 1% at a kinetic energy of 10 MeV. In all dynamic relations involving moving particles, the relativistic mass must be used. This is particularly important in the next chapter, where we discuss the effect of the acceleration of charged particles to high energies in nuclear particle accelerators.

12.2. The mass energy

As for radioactive decay, the energy of a nuclear reaction is given by its Q-value (3.5, 4.12):

$$Q \text{ (MeV)} = - 931.5 \, \Delta M^{\circ} \text{ (u)} \tag{12.2}$$

where

$$\Delta M^{\circ} = M_3^{\circ} + M_4^{\circ} - M_1^{\circ} - M_2^{\circ} \tag{12.3}$$

The reaction energy may thus be computed from the rest masses of the reactants and products. If mass disappears in the reaction ($\Delta M^{\circ} < 0$), energy is released: the reaction

is said to be *exoergic*, and Q is positive. For $Q < 0$ the reaction is *endoergic* and $\Delta M^{o} > 0$. (For comparison, in chemistry a negative value of ΔH is associated with exothermic spontaneous reactions.) Tables of Q-values, especially for light projectiles, can be found in several literature sources.

The relativistic mass equation (4.21)

$$E_{kin} = (m - m^{o})\mathbf{c}^{2} \tag{12.4}$$

can be separated into two terms if we define

$$E^{o}_{mass} \equiv m^{o}\,\mathbf{c}^{2} \tag{12.5}$$

and

$$E_{tot} \equiv m\,\mathbf{c}^{2} \tag{12.6}$$

Then

$$E_{tot} = E_{kin} + E^{o}_{mass} \tag{12.7}$$

We will call E^{o}_{mass} the *mass energy*, which is independent of the kinetic energy of the particle. E^{o}_{mass} is *potential* energy, and can in principle be converted into any other energy form because (12.5) is a form of Einstein's mass-energy relation. E^{o}_{mass} is closely related to the nuclear binding energy (cf. 3.4).

In the previous section we pointed out that for $Q > 0$ the reaction is exoergic and, as a consequence of that, mass has to disappear ($\Delta M^{o} < 0$). If the total energy of the system is constant (conservation rule (a)), it becomes obvious from (12.7) that when E^{o}_{mass} decreases E_{kin} must increase. Thus the products of the exoergic nuclear reaction have a higher kinetic energy than the reactants.

12.3. The Coulomb barrier

Equation (12.7) is a special case of a more general equation applicable to all systems of particles:

$$E_{tot} = E_{kin} + E_{pot} \tag{12.8}$$

where E_{kin} = translational, rotational, vibrational, etc., energy and E_{pot} = mass energy, gravitational, electrostatic energy, surface energy, chemical binding energy, etc. This is the total energy referred to in (4.2).

In nuclear reactions we would include the mass energy for the atomic masses in their ground state E^{o}_{mass}, the excitation energy of the nucleus above its ground state E_{exc}, the absorption or emission of photons in the reaction E_{υ}, and — in reactions between charged particles — the electrostatic potential (Coulomb) energy E_{coul}. Since we are concerned only with reactions induced by neutral or positively charged particles, the Coulomb energy is

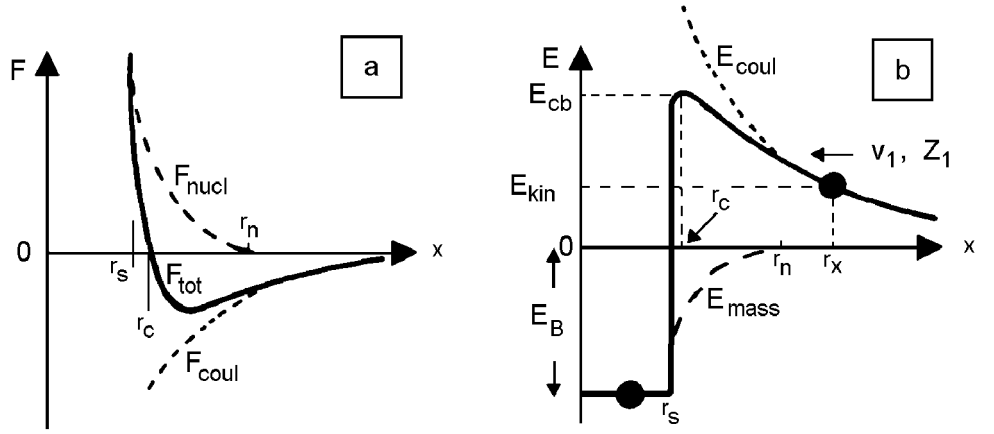

FIG. 12.2. Forces (a) and energy (b) conditions when a charged projectile (Z_1, v_1) reacts with a target nucleus.

zero or positive (i.e. repulsive). The incoming projectile must possess sufficient kinetic energy to overcome any repulsion. During the intermediate stage, this energy appears as partly as recoil energy of the center of mass and partly as potential energy. In the second step of the reaction the repulsion of charged products results in greater kinetic energy. Thus, generally,

$$E_{tot} = E_{kin} + E_{coul} + E^{o}_{mass} + E_{exc} + E_{\upsilon} \tag{12.9}$$

In nuclear reactions the total energy must be conserved, although the distribution of this energy in the different forms of (12.9) usually changes during the course of the reaction. For example, any decrease in the mass energy term must be balanced by a complimentary increase in one or more of the other energy terms. An example of this occurs in the explosion of nuclear weapons where a fraction of the rest mass is transformed to other forms of energy. The opposite reaction, the transformation of kinetic energy to mass, occurs in the production of elementary particles and in high energy acceleration of particles in cyclotrons, synchrotrons, etc.

Let us, as an example of the use of (12.9), consider a reaction of a positively charged particle (M_1, Z_1, v_1) with a target atom (M_2, Z_2, $v_2 \approx 0$). We make two simplifying assumptions: the target nucleus is in the center of the coordinate system (i.e. $v_i = 0$) and relativistic mass corrections can be neglected.

Because both projectile and target have positive charge they must repel each other according to the Coulomb law:

$$F_{coul} = k \, \mathbf{e} \, Z_1 \, \mathbf{e} \, Z_2 / x^2 \tag{12.10}$$

where k is 8.99×10^9 (N m^2 C^{-2}). This force is shown as a function of the distance between the particles in Figure 12.2a. At a distance greater than r_n only the Coulomb repulsive force is in operation; however, for distances less than r_n both the attractive nuclear force F_{nucl} and the repulsive Coulomb act upon the system. The total force is given by $F_{tot} = F_{coul} + F_{nucl}$. This is shown by a solid line in the Figure. At some particular

distance designated r_c, the forces balance each other and at shorter distances ($x < r_c$), the attractive nuclear force dominates. The distance r_c is known as the Coulomb radius; to be more exact, r_c is the sum of the projectile and target radii.

In Figure 12.2b the variation in the value of the different forms of (12.9) is indicated as a function of the distance x between the two particles. At long distances from the target nucleus the kinetic energy of the projectile is decreased due to the Coulomb repulsion. For such distances the nuclear force can be neglected and

$$E^{o}_{kin} = E_{kin} + E_{coul} \tag{12.11}$$

Where E^{o}_{kin} is the original kinetic energy of the projectile and E_{coul} is the electrostatic (Coulomb) potential energy, which at distance x is

$$E_{coul} = - k Z_1 Z_2 \mathbf{e}^2/x \tag{12.12}$$

As the projectile approaches the target nucleus, the Coulomb repulsion causes the potential energy to increase as the kinetic energy of the particle decreases. If this decrease in kinetic energy of the particle is such that the kinetic energy reaches a value of zero at any distance greater than r_c, the particle is reflected away from the nucleus before it is close enough to experience the attractive nuclear force. The projectile is thus hindered by a Coulomb potential barrier from causing nuclear reaction. A necessary condition for charged projectiles to cause nuclear reactions is that E^{o}_{kin} exceeds the Coulomb barrier height E_{cb}:

$$E_{cb} = k Z_1 Z_2 \mathbf{e}^2/r_c \tag{12.13}$$

This equation is useful for determining the Coulomb radius ("distance of closest approach") of nuclei; for conservation of momentum see (12.14 and 12.15).

We learned in Chapter 3 that experimentally, the nuclear radius is given by $r = r_o A^{1/3} \times 10^{-15}$ m. Thus far we have discussed three different kinds of nuclear radius constants — r_s, r_c, and r_n. In addition we have treated the target and the projectile as points in space. Experiments have indicated that the values of r_o are approximately 1.1 for the radius r_s of constant nuclear density, 1.3 for the Coulomb radius r_c, and 1.4 for the nuclear radius r_n, which includes surface effects.

To calculate the value of the energy of the Coulomb barrier we can use a model in which the target nuclei and the projectile are just touching so that r_c is taken as the sum of the radii of the projectile and the target nucleus. Moreover, we must now consider that the center of mass is not immobile in the collision but has a certain kinetic energy determined by the conservation of momentum. With this model the Coulomb barrier energy is given by the equation

$$E_{cb(min)} = 1.109\,(A_1 + A_2)[Z_1 Z_2/\{A_2(A_1^{1/3} + A_2^{1/3})\}] \ \text{(MeV)} \tag{12.14}$$

$E_{cb(min)}$ is the minimum energy that a projectile of mass A_1 and charge Z_1 must have in order to overcome the Coulomb barrier of a target nucleus of mass A_2 and charge Z_2 in a central collision. For the reaction between an α-particle and the nucleus ^{14}N, $E_{cb(min)}$ has a value of 4.99 MeV; it is obvious that Rutherford's α-particles of 7.68 MeV from the

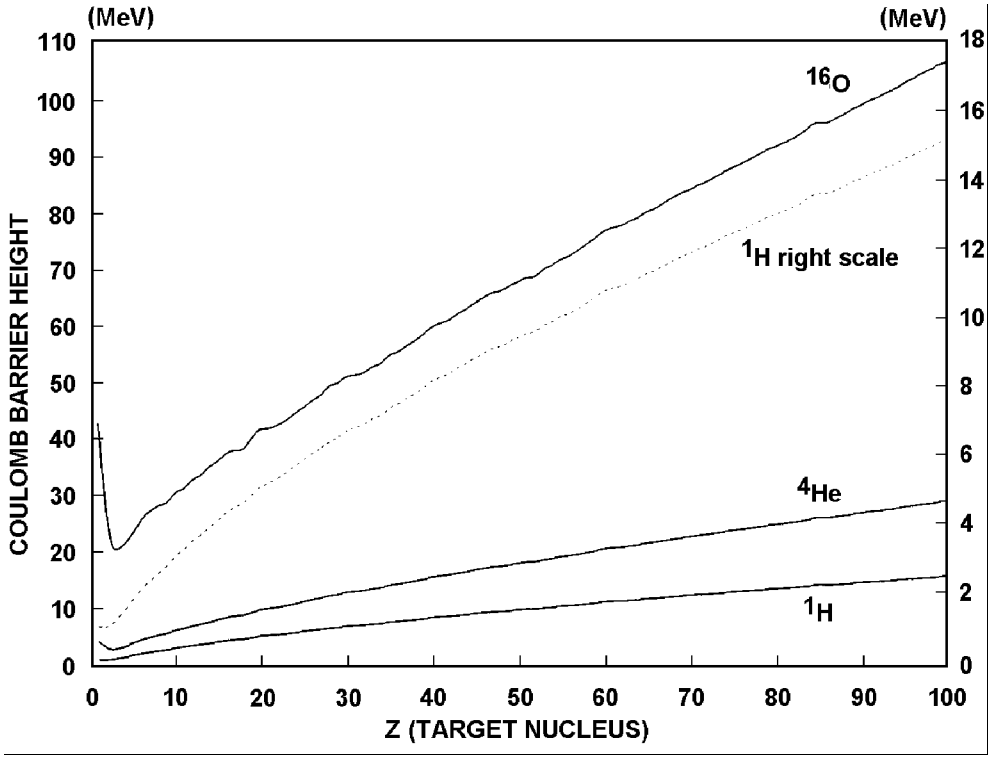

FIG. 12.3. The Coulomb barrier height, E_{cb}(min) according to (12.18) for reactions between a target element Z (of most common A) and projectiles ^{1}H, ^{4}He, and ^{16}O.

decay of ^{210}Po had sufficient kinetic energy to cause reaction.

In Figure 12.3 the values of $E_{cb(min)}$ for protons, α-, and ^{16}O-particles are shown as a function of the atomic number of the target nucleus. The A-values of the target are of the most stable isotope of the element Z. The initial decrease in the curves occur because in that range the projectile is heavier than the target nuclei.

In most cases the projectile-target collision is not head-on. Consider a projectile that would pass the target nucleus with a shortest distance, x, between their centers if no forces acted between them. This distance is called the *impact parameter*. The repulsive coulomb force will cause the projectile to pass at a larger distance. By considering conservation of momentum and energy, and assuming that the distance of closest approach, d, is outside the range of the nuclear force one obtains

$$d = r + [r^2 + x^2]^{½} \tag{12.15}$$

In this equation we have introduced the *collision radius*, r, defined as

$$r = \mathrm{k}\, Z_1\, Z_2\, \mathbf{e}^2 / [m_{\mathrm{red}}\, (v_1^0)^2] \tag{12.16}$$

where $\mathrm{k} = 8.99 \times 10^9$ N m^2 C^{-2}, $m_{\mathrm{red}}{}^{-1} = m_1^{-1} + m_2^{-1}$ and v_1^o is the initial projectile velocity in the laboratory coordinate system.

A technique called *Coulomb excitation* is used to induce rotationally excited states in nuclei; an example of this was shown in Figure 11.7. In order to impart high energy to the nucleus without causing nuclear transformation, heavy ions with kinetic energy below that required for passing over the Coulomb barrier ($E_1 < E_{\mathrm{cb(min)}}$) may be used as projectiles.

12.4. Rutherford scattering

If a collimated beam of particles (Z_1, A_1, $E_{\mathrm{kin},1}$) strikes a foil (Z_2, A_2) so that most of the particles pass through the foil without any reduction in energy, it is found that many particles are scattered away from their incident direction (Fig. 12.4). We can neglect multiple scattering and nuclear transformations, since they are many times less than the number of scattering events.

Geiger and Marsden found that scattering follows the relation

$$\mathrm{d}\sigma/\mathrm{d}\Omega = [k\, Z_1\, Z_2\, \mathbf{e}^2 / \{2 m_{\mathrm{red}} (v_1^o)^2\}]^2 / \sin^4(\theta/2) \tag{12.17}$$

Where $\mathrm{d}\sigma/\mathrm{d}\Omega$ is the differential cross-section (reaction cross-section in barns or m^2 per steradian), θ is the scattering angle (Fig. 12.4) and v_1^o incident particle velocity. The differential cross-section is a measure of the probability σ of a scattering event per unit solid angle Ω (1 steradian covers $1/(4\pi)$:th of the area of a sphere):

$$\mathrm{d}\sigma/\mathrm{d}\Omega = n/(n_o\, N_v\, x\, \Delta\Omega) \tag{12.18}$$

where n is the number of projectiles scattered into the detector which subtends a solid angle

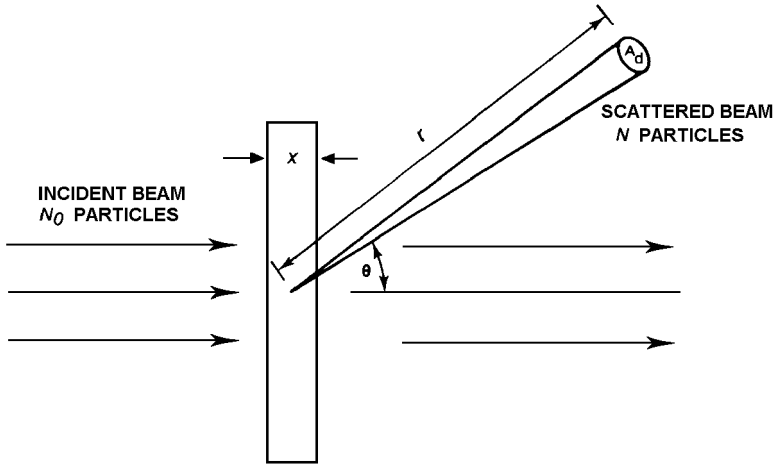

FIG. 12.4. Geometry for measuring the scattering by a thin foil.

$$\Delta\Omega = A_d\, r^{-2} \tag{12.19}$$

with respect to the center of the target; A_d is the detector area and r is the distance between target and detector; n_o is the number of projectiles hitting the target, which is x thick and contains N_v scattering atoms per unit volume ($N_v = \varrho \mathbf{N}_A/M$, where ϱ is the density).

By bombarding foils made of different metals with α-particles from radioactive elements, (12.17) can be used to prove that Z_2 is identical to the atomic number of the metal. This implies that the atom has a central core with a positive charge of that magnitude. Such experiments showed that some particles scattered almost directly back (i.e. $\theta > 90°$), a fact which baffled Rutherford: "It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you fired a fifteen inch shell at a piece or tissue paper and it came back and hit you." From the scattering equations he concluded that the scattering center, the "nucleus", had a diameter which was 1000 times smaller than the atom. This same scattering technique was used 55 years later in the US lunar explorer Surveyor 5 (1967) to determine the composition of the lunar surface.

12.5. Elastic scattering

In elastic scattering energy is exchanged between the projectile and the target nucleus but the value of Q is zero:

$$X_1(v_1) + X_2(v_2 \approx 0) \rightarrow X_1(v_1') + X_2(v_2' > 0) \qquad Q = 0 \tag{12.20}$$

From the conservation of momentum and energy one finds that for a central collision:

$$v_1' = v_1(M_1 - M_2)/(M_1 + M_2);\ v_2' = 2v_1M_1/(M_1 + M_2) \tag{12.21}$$

v_1 is the projectile velocity, and v_1' and v_2' are the velocities of the scattered species.

An important elastic scattering reaction in nuclear reactors involves the slowing down of neutrons from high kinetic energies, which they possess when emitted in nuclear fission, to very low energies at which they have much higher reaction probabilities with the nuclear fuel. The neutrons are slowed to energies comparable to those of a neutron gas at the temperature of the material in which they are moving and, hence, they are known as *thermal neutrons*. The most probable kinetic energy for particles at thermal equilibrium at the temperature T is given by the relation $E_{kin} = \mathbf{k}T$. At 25°C the most probable energy for thermal neutrons is 0.026 eV, while the average energy is approximately 0.040 eV. In nuclear reactions it is necessary to consider the most probable kinetic energy rather than the average energy (cf. §2.6.2).

Equation (12.21) shows that for the case of head-on (or central) collisions between energetic neutrons and protons at thermal energies ($v_1 \approx 0$), the velocity of the neutron is reduced in a single step to thermal energy as a result of the approximately equal masses of the neutron and proton. This effect can be seen in a head-on collision between equally heavy billiard balls. If the target atom is ^{12}C, the velocity of a neutron decreases approximately 15% in a single collision, while the energy of the neutron decreases 28%. If the target atoms are as heavy as uranium, the decrease in velocity and in kinetic energy

TABLE 12.1. *Distance from neutron source for maximum thermal flux*

Neutron source	Neutron energy (at source)	Moderator	
		H_2O	D_2O
RaBe	Average 4 MeV	10 cm	
TD reaction	14 MeV	15 cm	
Fission	Average 2 MeV	7 cm	21 cm

is on the order of about 1% per collision.

The slowing down of energetic neutrons to low kinetic energies is called *moderation*. Obviously, the lightest atoms are the best *moderators* for fast neutrons and heavier atoms are poorer moderators. Conversely, heavier atoms will be better *reflectors* for energetic neutrons since they will scatter them back with little loss in energy. For thermal neutrons, however, heavy atoms are not good reflectors because of their greater tendency to absorb the neutrons. Noncentral collisions ($\theta > 0$) are more common than central ones, so the average decrease in neutron energy per collision is less than the values calculated for central collisions (see further Ch. 19).

In a point neutron source, surrounded by a moderator, the thermal neutron flux (particles per unit area and unit time) as a function of the distance from the source is found initially to increase and then to decrease; see Fig. 12.5. The distance from the source at which the flux is at maximum is given in Table 12.1. This is the optimum position for thermal neutron induced reactions.

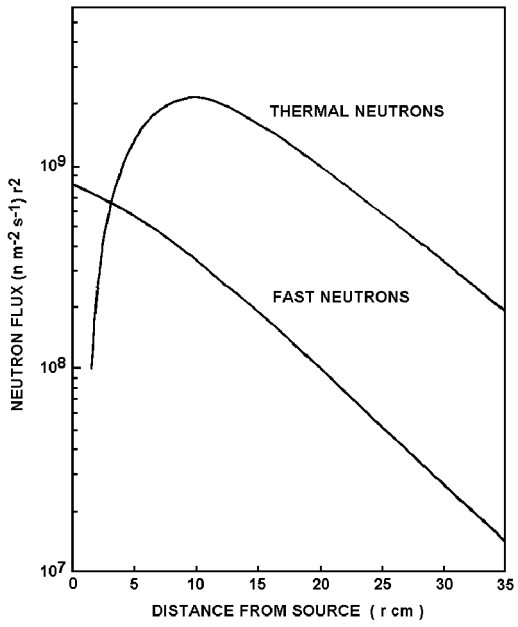

FIG. 12.5. The thermalization in water of fast neutrons from a RaBe point neutron source (intensity 10^6 n s^{-1}).

12.6. Inelastic scattering

In the class of nuclear reactions termed inelastic scattering, part of the kinetic energy of the projectile is transferred to the target nucleus as excitation energy without changing the values of *A* or *Z* of either target or projectile. In case the projectile is a heavy ion it may also become excited. However, the collision of the projectile and target forming the products does result in a value of *Q* different than zero. For light projectiles the process can be written:

$$X_1 + X_2 \begin{cases} a \rightarrow X_1 + X_2 + \gamma & (12.22a) \\ b \rightarrow X_1 + X_2^* & (12.22b) \end{cases} \qquad Q < 0$$

The reaction path *a* indicates that the energy *Q* is emitted as a γ-ray. In reaction path *b* the *Q* is retained as excitation energy of the target nuclide. The latter exists in an excited state and may transform to the ground state quite rapidly or may exist for a measurable time as an isomer. An example of inelastic scattering reaction (12.22b) is the formation of an isomer of silver by the irradiation of ^{107}Ag with neutrons

$$^{107}\text{Ag(n,n')}^{107\text{m}}\text{Ag} \rightarrow {}^{107}\text{Ag} + \gamma$$

the half-life of $^{107\text{m}}$Ag is 44 s. The energy relationships in inelastic scattering are the same as for other nuclear reactions.

12.7. Dissecting a nuclear reaction

As an example of a nuclear transmutation reaction, let us consider the following:

$$^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow [{}^{18}_{9}\text{F}]^* \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$$

For pedagogic reasons we treat this reaction as if an intermediate *compound nucleus* is formed. The compound nucleus $^{18}_{9}$F is in square brackets to indicate its transitory nature and marked with an asterisk to indicate that it is excited. Induced nuclear reactions are often written in an abbreviated manner indicating first the target and then, in parentheses, the projectile and the smaller product, followed by the major product outside the parentheses. In the case of the sample reaction, we would write ^{14}N(α,p)^{17}O The abbreviations used for ^{4}He, ^{1}H, ^{2}H (= D), ^{3}H (= T), etc., are α, p, d, t, etc. The reactions may be classified by the particles in parentheses; the sample reaction is called an (α,p) type.

The sample reaction is of historical interest since this is actually the reaction studied by Rutherford in 1919 when he produced the first induced nuclear transformation in the laboratory. A cloud chamber photograph of the reaction is shown in Figure 12.6.

From the atomic rest masses we can calculate that the change in mass for this reaction is

$$\Delta M^{\circ} = (M_3^{\circ} + M_4^{\circ} - M_1^{\circ} - M_2^{\circ}) = (16.999\,131 + 1.007\,825 - 14.003\,074 - 4.002\,603) =$$

$$= 0.001\,279 \text{ (u)}$$

This increase in mass corresponds to a Q-value of -1.19 MeV (12.2). The energy required for this endoergic reaction can only be obtained through the kinetic energy of the projectile.

In the collision between the projectile and the target nucleus, which we have assumed to be stationary, the compound nucleus formed always acquires a certain kinetic energy, recoil, which can be calculated from the conservation laws to be

$$E_{\text{kin,c}} = E_{\text{kin,1}} M_1/(M_1 + M_2) \approx E_{\text{kin,1}} A_1/A_{\text{c}} \tag{12.23}$$

The mass numbers can be substituted for the atomic masses since we are carrying the calculation of the energy to three significant figures only. In order for the reaction to occur, the kinetic energy of the projectile $E_{\text{kin,1}}$ must exceed the kinetic energy of the compound nucleus $E_{\text{kin,c}}$ by the value of Q:

$$E_{\text{kin,1}} = E_{\text{kin,c}} - Q \tag{12.24}$$

The minimum projectile energy necessary for a reaction to occur is called the *threshold energy* E_{tr}, which according to (12.23) and (12.24) is

$$E_{\text{tr}} = -Q(M_1 + M_2)/M_2 \approx -QA_{\text{c}}/A_2 \quad (\text{if } Q < 0) \tag{12.25}$$

For our sample reaction, the threshold energy is $1.19 \times 18/14 = 1.53$ MeV.

If the projectile has a higher kinetic energy than the threshold energy, the products would have a correspondingly higher combined kinetic energy, assuming no transformation into

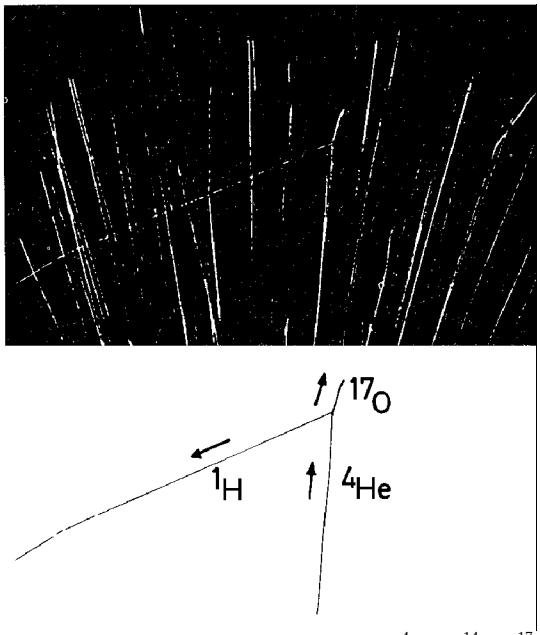

FIG. 12.6. Cloud chamber photograph of the Rutherford reaction $^4\text{He} + {}^{14}\text{N} \rightarrow {}^{17}\text{O} + {}^1\text{H}$. (From Blackett and Lees.)

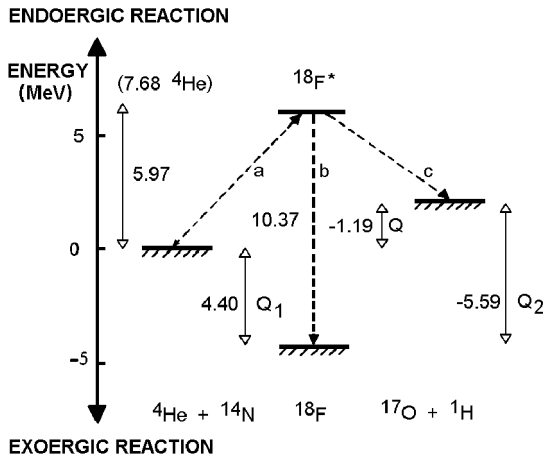

FIG. 12.7. Energy diagram of the transmutation $^{4}He + {}^{14}N \rightarrow {}^{18}F^{*} \rightarrow {}^{17}O + {}^{1}H$ caused by bombarding ^{14}N with 7.68 MeV α-particles from ^{214}Po.

other forms of energy. Rutherford used α-particles from the radioactive decay of ^{214}Po, which have kinetic energies of 7.68 MeV. The products, therefore, had considerable kinetic energy, which is shown by the thick tracks in the cloud chamber picture of Figure 12.6.

In Figure 12.7 the energy pattern for our reaction is summarized. The *Q*-value for the first step of the reaction ($^{4}He + {}^{14}N \rightarrow {}^{18}F$) is +4.40 MeV ($Q_1$). The kinetic energy of the ^{4}He is 7.68 MeV, so conservation of momentum requires that the ^{18}F nucleus have $7.68 \times 4/(4+14) = 1.71$ MeV of kinetic energy. Thus, the internal excitation energy of $[^{18}F]^{*}$ is $4.40 + 7.68 - 1.71 = 10.37$ MeV. For the second step, $^{18}F \rightarrow {}^{17}O + {}^{1}H$, the Q_2-value is −5.59 MeV. This shows that the total excitation and kinetic energy of the products is $10.37 + 1.71 - 5.59 = 6.49$ MeV.

12.8. The compound nucleus model

We have used a model which assumes the formation of an intermediary excited nucleus in the previous section to discuss some aspects of nuclear reactions. Let us now study this model a little closer.

If $E^{o}_{kin} > E_{cb}$ the attractive nuclear force dominates and the particle is absorbed by the target nucleus. Assuming $Q > 0$, the E^{o}_{mass} decreases. This means that E_{exc} increases, and the system is transformed into an excited compound nucleus. Moreover, since the projectile had a certain kinetic energy, the excitation of the compound nucleus is

$$E_{exc} = Q + E^{o}_{kin} \tag{12.26}$$

This equation is based on the assumption that the compound nucleus is the center of reference. It should be noted that the height of the Coulomb barrier does not influence the excitation energy of the nucleus in any way other than that a projectile must have a kinetic energy greater than that value before the reaction can occur.

In the reaction all E_{kin} and E_{coul} are transferred into E_{kin}, E_{exc} and E^{o}_{mass} for the compound nucleus. The excitation of the compound nucleus can be removed either through the emission of γ-rays, neutrons, or by the decay of the compound nucleus into different product nuclei. In the former case, $E_{kin} = E_{\upsilon}$. In the latter case we again have to bring in (12.9).

If the reaction passes through formation of a compound nucleus, having a lifetime of many nuclear vibrations, we should expect that the amounts and types of reaction products were independent of the projectile — target combination used to produce a specific compound nucleus and only depend on its excitation energy. This has indeed been found to be the case for some kinds of nuclear reactions, see Ch. 14.

12.9. Radioactive neutron sources

Neutrons were discovered through the reaction between α-particles, emitted by radioactive substances, and light elements: ^{4}He (from Po) + ^{9}Be → ^{12}C + n (Chadwick 1932). The coulomb barrier of light elements is sufficiently small to be penetrated by α-particles emitted by radioactive nuclides; in the reaction used by Chadwick the coulomb barrier is only ~ 3.5 MeV, eqn. (12.14) and Fig. 12.3. All early neutron research was conducted with sources of this kind, the most popular being the RaBe mixture. A number of radioactive n-sources are listed in Table 12.2.

The nuclide $^{252}_{98}$Cf emits neutrons through spontaneous fission in ~ 3% of all decays, the rest being α-decays. All the other neutron sources listed involve a radioactive nuclide whose decay causes a nuclear reaction in a secondary substance which produces neutrons. For example, $^{124}_{51}$Sb produces neutrons in beryllium powder or metal as a result of the initial emission of γ-rays, in which case there is no coulomb barrier to penetrate. Radium, polonium, plutonium, and americium produce neutrons by nuclear reactions induced in beryllium by the α-particles from their radioactive decay. For the neutrons produced either by spontaneous fission in californium or by the α-particle reaction with beryllium, the

TABLE 12.2. *Radioactive neutron sources*

Material	Half-life	Neutron yield ($n\ s^{-1}\ Bq^{-1}$)	γ-dose rate[a] ($mSv\ h^{-1}\ m^{2}\ Bq^{-1}$)
^{226}Ra + Be	1600 y	3.5×10^{-4}	850
^{239}Pu + Be	24110 y	2.4×10^{-4}	4
^{239}Pu + ^{18}O	24110 y	$\leq 7.8 \times 10^{-6}$	[b]
^{241}Am + Be	433 y	6.8×10^{-5}	2.5
^{210}Po + Be	138 d	6.8×10^{-5}	< 0.3[c]
^{124}Sb + Be	60 d	$\sim 5 \times 10^{-6}$	1000
^{252}Cf	2.6 y	$5 \times 10^{12}\ n\ s^{-1}\ g^{-1}$	[d]

[a] Dose rate at 1 m from source, cf. eqn. (7.9). [b] Variable source of $Pu^{16}O_2 + H_2{}^{18}O$; by transferring water through heating into the porous oxide, the reaction $^{18}O(\alpha,n)^{21}Ne$ is initiated; by cooling the water evaporates and is returned to storage whereby the n output decreases to < 0.01 of maximum output. [c] The half-life can be increased to 22 y by including the precursor nuclide ^{210}Pb. [d] γ-dose rate increases with time due to formation of γ-emitting daughters.

energy is between 0 and 10 MeV, while for the neutrons emitted in the γ,n-reaction involving ^{124}Sb, the energy is 0.02 MeV. Neutron sources are commercially available.

Neutron multipliers have also come into use. These usually consist of a tank containing highly enriched ^{235}U. The uranium releases 100 n for each trigger neutron sent in; the common source is ^{252}Cf. These units are used for neutron radiography (§6.9.2), activation analysis (§9.3.3), etc.

Although the neutron flux from radioactive sources is comparatively small, they are still quite useful for specific purposes, due to their extreme simplicity, reliability (they cannot be shut off), and small size. The γ-radiation which is also present is, however, a disadvantage. From this point of view, ^{252}Cf is the most preferable neutron source, but, on the other hand, it has a relatively short half-life compared with some of the other sources. ^{252}Cf is further discussed in Chapter 16.

12.10. Exercises

12.1. (a) What kinetic energy must be given to a helium atom in order to increase its mass by 1%? (b) What are the mean velocity and the mean kinetic energy of a helium atom at STP?

12.2. Calculate the distance of closest approach for 5 MeV α-particles to a gold target.

12.3. In a Rutherford scattering experiment ^{2}H atoms of 150 keV are used to bombard a thin ^{58}Ni foil having a surface density of 67×10^{-6} g cm^{-2}. The detector subtends a solid angle of 1.12×10^{-4} sr and detects 4816 deuterons out of a total of 1.88×10^{12} incident on target. Calculate (a) the differential cross-section (in barns). (b) What is the distance between target and the solid state detector, which has a surface area of 0.2 cm^2?

12.4. In Rutherford scattering on a silver foil using α-particles from a thin-walled radon tube, the following data were observed: $d\sigma/d\Omega$ = 22(θ 150°), 47(105°), 320(60°), 5260(30°), 105 400(15°) barns per steradian. Calculate the energy of the incident α-particles.

12.5. Alpha-particles from ^{218}Po (E_α 6.0 MeV) are used to bombard a gold foil. (a) How close to the gold nucleus can these particles reach? (b) What is the nuclear radius of gold according to the radius-mass relation (r_o = 1.3 fm)?

12.6. What is the *Q*-value for the reactions: (a) ^{11}B(d,α)^{9}Be; (b) ^{7}Li(p,n)^{7}Be?

12.7. What is the maximum velocity that a deuteron of 2 MeV can impart to a ^{16}O atom?

12.8. Calculate the mass of an electron accelerated through a potential of 2×10^8 V.

12.9. ^{12}C atoms are used to irradiate ^{239}Pu to produce an isotope of berkelium. What is the Coulomb barrier height?

12.10. Measurements made on the products of the reaction ^{7}Li(d,α)^{5}He have led to an isotopic mass of 5.0122 for the hypothetical nuclide ^{5}He. Show that this nuclear configuration cannot be stable by considering the reaction ^{5}He → ^{4}He + n.

12.11. In an experiment one hopes to produce the long-lived (2.6 y) ^{22}Na through a d,2n-reaction on neon. What is (a) the *Q*-value, (b) the threshold energy, (c) the Coulomb barrier height, and (d) the minimum deuteron energy for the reaction? The mass excesses (in keV) are −5185 for ^{22}Na and −8027 for ^{22}Ne.

12.11. Literature

K. S. KRANE, *Introductory Nuclear Physics*, J. Wiley & Sons, 1988.
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