Particle Interactions

Heavy Particle Collisions

- Charged particles interact in matter.
  - Ionization and excitation of atoms
  - Nuclear interactions rare
- Electrons can lose most of their energy in a single collision with an atomic electron.
- Heavier charged particles lose a small fraction of their energy to atomic electrons with each collision.

Energy Transfer

- Assume an elastic collision.
  - One dimension
  - Moving particle M, V
  - Initial energy $E = \frac{1}{2} MV^2$
  - Electron mass m
  - Outgoing velocities V_f, v_f

- This gives a maximum energy transfer Q_{max}.

$$MV = MV_f + mv_f$$

$$\tfrac{1}{2} MV^2 = \tfrac{1}{2} MV_f^2 + \tfrac{1}{2} mv_f^2$$

$$\Downarrow$$

$$V_f = \frac{M-m}{M+m} V$$

$$Q_{\max} = \tfrac{1}{2} MV^2 - \tfrac{1}{2} MV_f^2$$

$$= \frac{4mM}{(M+m)^2} \left(\tfrac{1}{2} MV^2\right) \cong \frac{4mE}{M}$$

Relativistic Energy Transfer

- At high energy relativistic effects must be included.

$$Q_{\max} = \frac{2\gamma^2 mV^2}{1 + 2\gamma m / M + m^2 / M^2}$$

- This reduces for heavy particles at low γ, $\gamma m/M << 1$.

$$Q_{\max} \cong 2\gamma^2 mV^2 = 2\gamma^2 \beta^2 mc^2$$

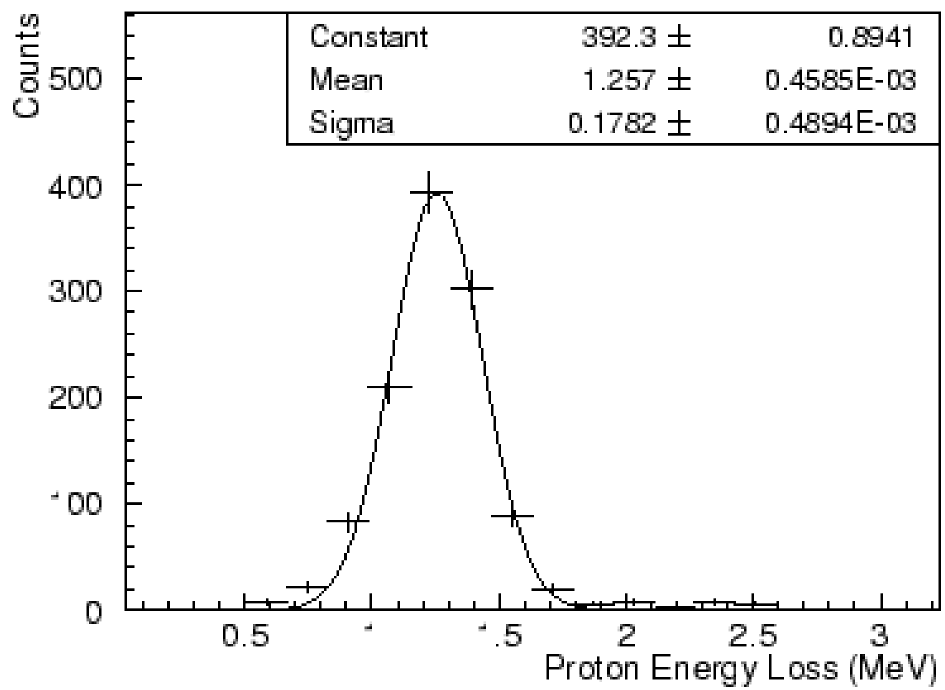
$$Q_{\max} \cong 2(\gamma^2 - 1)mc^2$$

Typical Problem

- Calculate the maximum energy a 100-MeV pion can transfer to an electron.
- m_π = 139.6 MeV = 280 m_e
  - problem is relativistic
- $\gamma = (K + m_\pi)/ m_\pi = 2.4$
- $Q_{\max}$ = 5.88 MeV

Protons in Silicon

- The MIDAS detector measured proton energy loss.
  - 125 MeV protons
  - Thin wafer of silicon



MIDAS detector 2001

Linear Energy Transfer

- Charged particles experience multiple interactions passing through matter.
  - Integral of individual collisions
  - Probability *P*(*Q*) of energy transfer Q

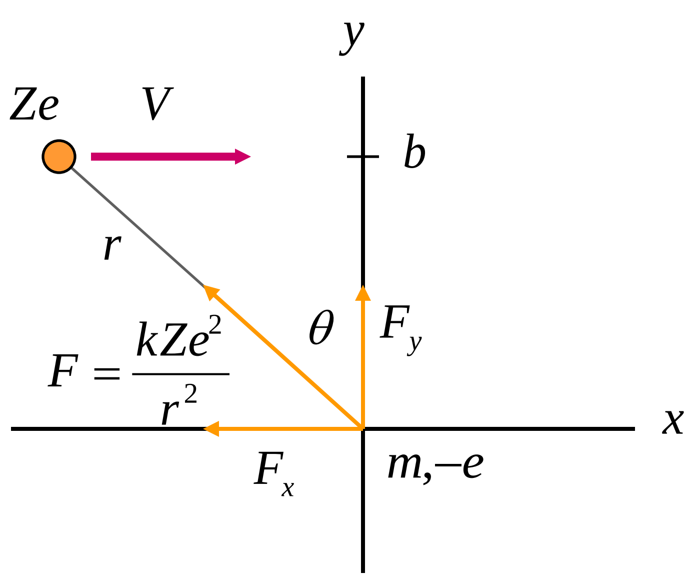
$$\overline{Q} = \int_{Q_{\min}}^{Q_{\max}} QP(Q)dQ$$

- Rate of loss is the *stopping power* or *LET* or *dE*/*dx*.
  - Use probability of a collision μ (cm^{-1})

$$-\frac{dE}{dx} = \mu\overline{Q} = \mu\int_{Q_{\min}}^{Q_{\max}} QP(Q)dQ$$

Energy Loss

- The stopping power can be derived semiclassically.
  - Heavy particle *Ze*, *V*
  - Impact parameter *b*



- Calculate the impulse and energy to the electron.

$$p = \int F_y dt = \int \frac{kZe^2}{r^2} \frac{b}{r} dt$$

$$p = kZe^2 \int \frac{b}{(b^2 + V^2 t^2)^{3/2}} dt$$

$$p = \frac{2kZe^2}{Vb}$$

$$Q = \frac{p^2}{2m} = \frac{2k^2 Z^2 e^4}{mV^2 b^2}$$

Impact Parameter

- Assume a uniform density of electrons.
  - n per unit volume
  - Thickness dx
- Consider an impact parameter range b to $b+db$
  - Integrate over range of b
  - Equivalent to range of Q

- Find the number of electrons.

$$2\pi nb(db)dx$$

- Now find the energy loss per unit distance.

$$-\frac{dE}{dx} = 2\pi n\int Qbdb = \frac{4\pi nk^2Z^2e^4}{mV^2}\int\frac{db}{b}$$

$$-\frac{dE}{dx} = \frac{4\pi nk^2Z^2e^4}{mV^2}\ln\frac{b_{\max}}{b_{\min}}$$

Stopping Power

- The impact parameter is related to characteristic frequencies.
- Compare the maximum b to the orbital frequency f.
  - $b/V < 1/f$
  - $b_{\text{max}} = V/f$
- Compare the minimum b to the de Broglie wavelength.
  - $b_{\text{min}} = h / mV$

- The classical Bohr stopping power is

$$-\frac{dE}{dx} = \frac{4\pi n k^2 Z^2 e^4}{mV^2} \ln \frac{mV^2}{hf}$$

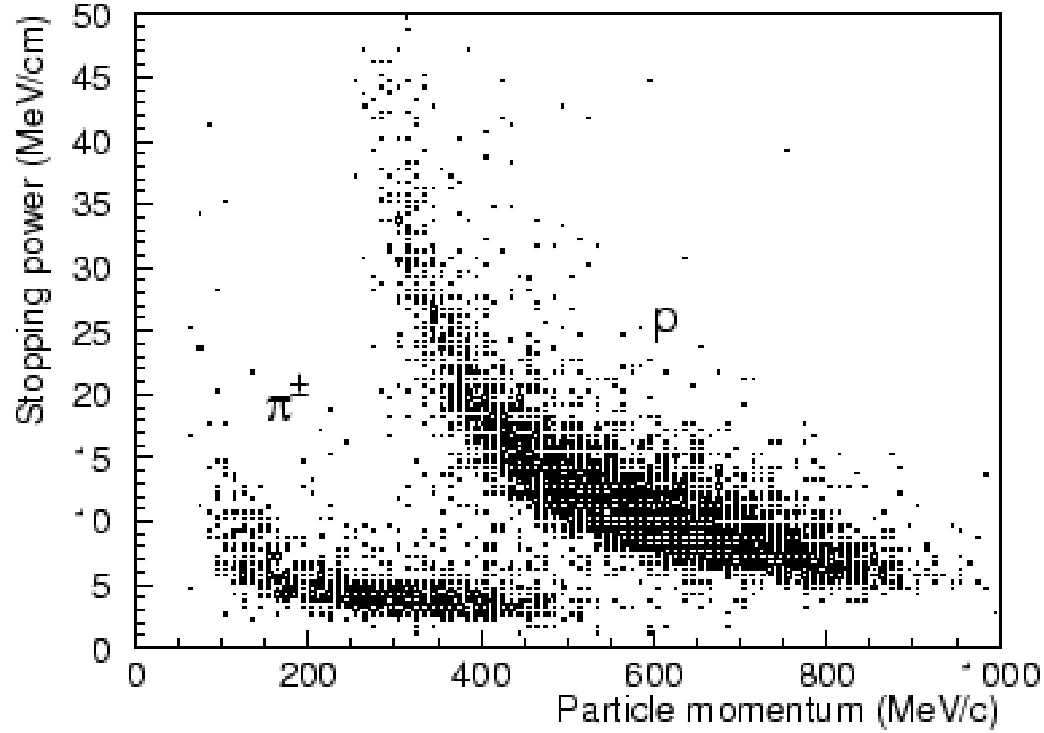
Bethe Formula

- A complete treatment of stopping power requires relativistic quantum mechanics.
  - Include speed b
  - Material dependent excitation energy

$$-\frac{dE}{dx}=\frac{4\pi nk^2Z^2e^4}{mc^2\beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)}-\beta^2\right]$$

Silicon Stopping Power

- Protons and pions behave differently in matter
  - Different mass
  - Energy dependent



MIDAS detector 2001

Range

- Range is the distance a particle travels before coming to rest.
- Stopping power represents energy per distance.
  - Range based on energy
- Use Bethe formula with term that only depends on speed
  - Numerically integrated
  - Used for mass comparisons

$$R(K) = \int_0^K \left(-\frac{dE}{dx} \right)^{-1} dE$$

$$R(K) = \frac{1}{Z^2} \int_0^K \frac{dE}{G(\beta)}$$

$$R(\beta) = \frac{1}{Z^2} \int_0^\beta \frac{Mg(\beta)d\beta}{G(\beta)} = \frac{M}{Z^2} f(\beta)$$

Alpha Penetration

Typical Problem

- Part of the radon decay chain includes a 7.69 MeV alpha particle. What is the range of this particle in soft tissue?

- Use proton mass and charge equal to 1.
  - $M_\alpha = 4$, $Z^2 = 4$

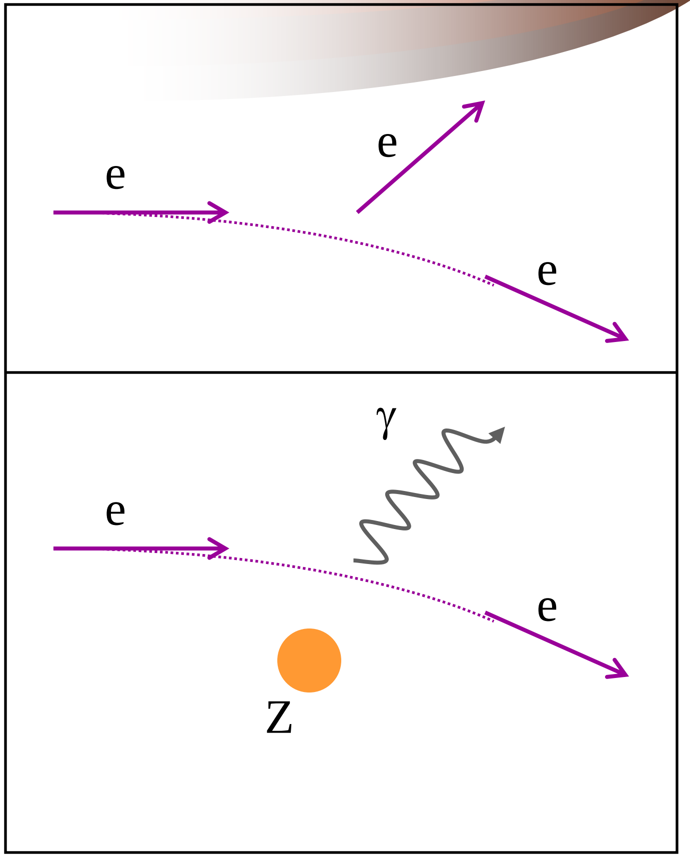
$$R_\alpha(\beta) = \frac{M}{Z^2} R_p(\beta) = R_p(\beta)$$

- Equivalent energy for an alpha is ¼ that of a proton.
  - Use proton at 1.92 MeV
- Approximate tissue as water and find proton range from a table.
  - 2 MeV, $R_p = 0.007$ g cm^{-2}

- Units are density adjusted.
  - $R_\alpha = 0.007$ cm
- Alpha can't penetrate the skin.

Electron Interactions

- Electrons share the same interactions as protons.
  - Coulomb interactions with atomic electrons
  - Low mass changes result
- Electrons also have stopping radiation: bremsstrahlung
- Positrons at low energy can annihilate.



Beta Collisions

- There are a key differences between betas and heavy ions in matter.
  - A large fractional energy change
  - Indistinguishable β^- from e in quantum collision

- Bethe formula is modified for betas.

$$\left(-\frac{dE}{dx}\right)^{\pm}_{\text{col}} = \frac{4\pi n k^2 e^4}{mc^2\beta^2}\left[\ln\frac{2mc^2\tau\sqrt{\tau+2}}{\sqrt{2}I} - F^{\pm}(\beta)\right] \qquad \tau = \frac{K}{mc^2}$$

$$F^-(\beta) = \frac{1-\beta^2}{2}\left[1+\frac{\tau^2}{8}-(2\tau+1)\ln 2\right]$$

$$F^+(\beta) = \ln 2 - \frac{\beta^2}{24}\left[23+\frac{14}{\tau+2}+\frac{10}{(\tau+2)^2}+\frac{4}{(\tau+2)^3}\right]$$

Radiative Stopping

- The energy loss due to bremsstrahlung is based classical electromagnetism.
  - High energy
  - High absorber mass

$$\left(-\frac{dE}{dx}\right)_{\text{rad}} = \frac{4nk^2Z(Z+1)e^4\tau}{137mc^2}\left[\ln 2\tau - \frac{1}{3}\right]$$

- There is an approximate relation.

$$\left(\frac{dE}{dx}\right)_{\text{rad}} = \frac{KZ}{700}\left(\frac{dE}{dx}\right)^{\pm}_{\text{col}}$$

-(dE/dx)/ρ in water

	MeV cm^2 g^{-1}	
Energy	col	rad
1 keV	126	0
10 keV	23.2	0
100 keV	4.2	0
1 MeV	1.87	0.017
10 MeV	2.00	0.183
100 MeV	2.20	2.40
1 GeV	2.40	26.3

Beta Range

- The range of betas in matter depends on the total *dE*/*dx*.
  - Energy dependent
  - Material dependent
- Like other measures, it is often scaled to the density.

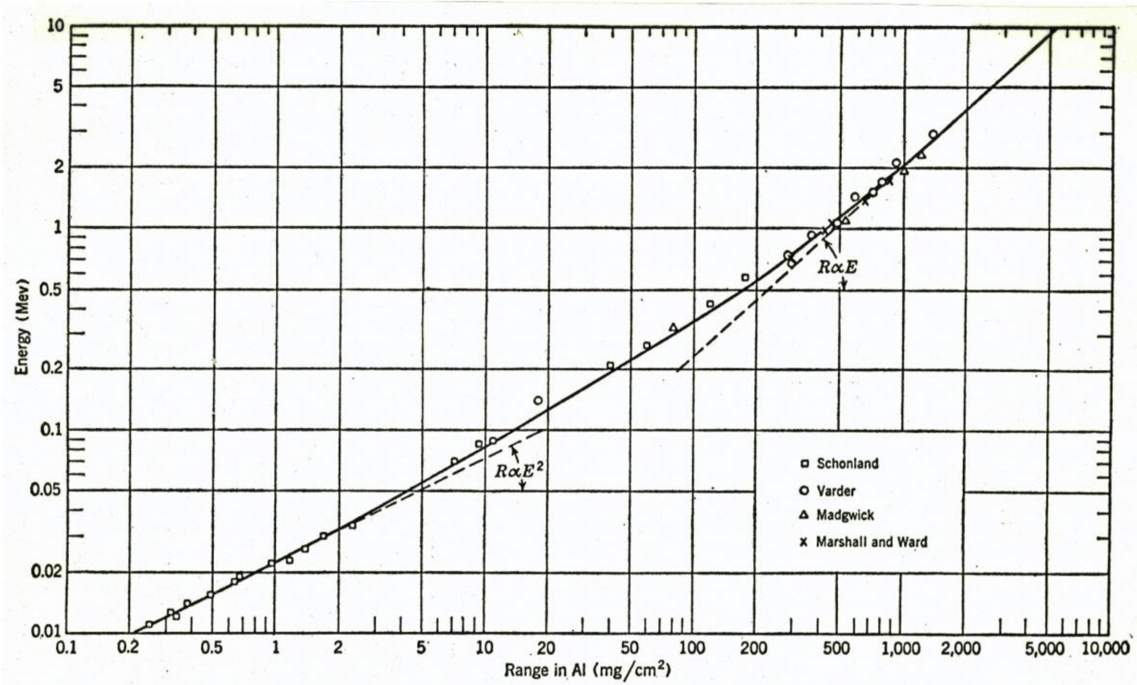


Fig. 3.3 Empirical range-energy relationship for electrons absorbed in aluminum. Experimental values by several observers (S16, V4, M4, M17) on monoenergetic electrons are shown. For *monoenergetic electrons*, the range coordinate refers to the *extrapolated range* R_0 of Fig. 3.2. For *continuous β-ray spectra* the energy coordinate refers to the end-point energy E_{max}, and the range coordinate becomes the *maximum range* R_m of Fig. 3.4. The smooth curve represents the empirical relationship, Eqs. (3.3) and (3.4), developed by Katz and Penfold (K7).

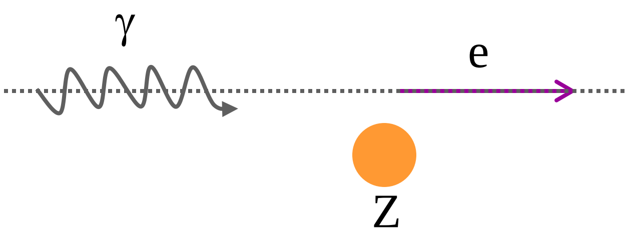
Photon Interactions

- High energy photons interact with electrons.
  - Photoelectric effect
  - Compton effect

- They also indirectly interact with nuclei.
  - Pair production

Photoelectric Effect

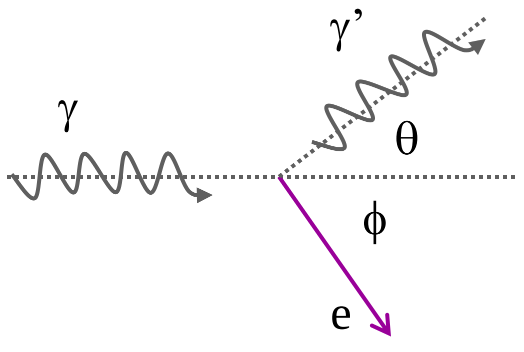
- A photon can eject an electron from an atom.
  - Photon is absorbed
  - Minimum energy needed for interaction.
  - Cross section decreases at high energy



$$K_e = h\nu - \varphi$$

Compton Effect

- Photons scattering from atomic electrons are described by the Compton effect.
  - Conservation of energy and momentum



$$h\nu + mc^2 = h\nu' + E'$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + P'\cos\phi$$

$$\frac{h\nu'}{c}\sin\theta = P'\sin\phi$$

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{mc^2}(1-\cos\theta)}$$

Compton Energy

- The frequency shift is independent of energy.
- The energy of the photon depends on the angle.
  - Max at 180°
- Recoil angle for electron related to photon energy transfer
  - Small $\theta \rightarrow$ cot large
  - Recoil near 90°

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$

$$K = \frac{h\nu(1-\cos\theta)}{mc^2/h\nu + 1 - \cos\theta}$$

$$\cot\frac{\theta}{2} = \left(1+\frac{h\nu}{mc^2}\right)\tan\phi$$

Compton Cross Section

- Differential cross section can be derived quantum mechanically.
  - Klein-Nishina
  - Scattering of photon on one electron
  - Units m^2 sr^{-1}

- Integrate to get cross section per electron
  - Multiply by electron density
  - Units m^{-1}

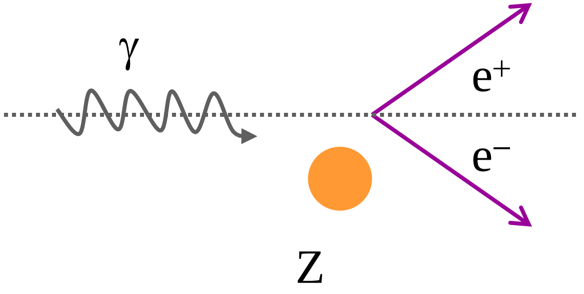
$$\frac{d\sigma}{d\Omega} = \frac{k^2 e^4}{2m^2 c^4}\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - \sin^2\theta\right)$$

$$\sigma = 2\pi n \int \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$$

Pair Production

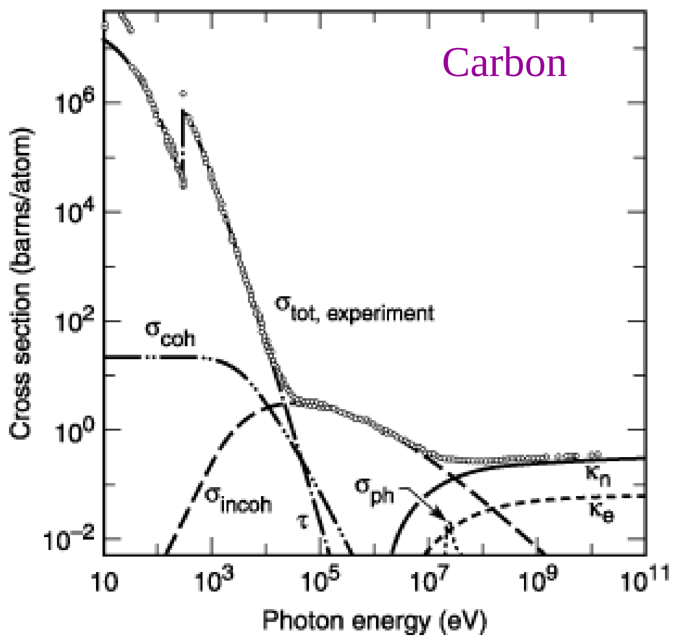
- Photons above twice the electron rest mass energy can create a electron positron pair.
  - Minimum λ = 0.012 Å
- The nucleus is involved for momentum conservation.
  - Probability increases with Z
- This is a dominant effect at high energy.

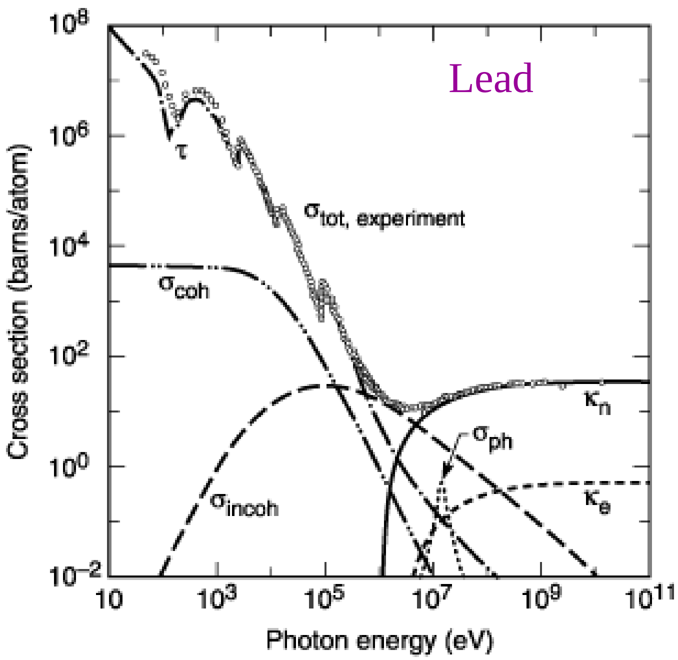
$$h\nu = 2mc^2 + K_+ + K_-$$



Total Photon Cross Section

- Photon cross sections are the sum of all effects.
  - Photoelectric τ, Compton σ_{incoh}, pair κ





J. H. Hubbell (1980)