Why is the bubble-density higher for slow tracks?

* INTRODUCTION

When a massive ($m>>m_e$) charged particle travels through matter (a bubble chamber liquid, for example), it exerts Coulomb forces on the electrons of the atoms, imparting momentum to these electrons.

In high energy physics we deal with particles of energies from a few hundred MeV to hundreds of GeV – enormous compared to the few eV needed to ionize atoms.

From $E^2 = p^2c^2 + m^2c^4$ we see that when the momentum (in GeV/c) become more than a few times the rest mass (in GeV/c^2), the particle is relativistic: $E \sim pc$. In our bubble chamber pictures, such relativistic particles are `minimum ionizing'.

Exercise: Many of our bubble chamber pictures involve a K^- beam with an energy of 4.2 GeV. What is the velocity of a 4.2 GeV kaon? Would it be minimum ionizing?
Answer:
By the following calculation:

$$m_K c^2 \approx 0.5\,\text{GeV} \Rightarrow m_K{}^2 c^4 \approx 0.25\,\text{GeV}^2$$
$$E_K^2 \approx (4.2\ \text{GeV})^2 = 17.6\,\text{GeV}^2$$
$$\therefore\ p_K^2 c^2 = E_K^2 - m_K{}^2 c^4 = 17.35\,\text{GeV}^2 >> m_K{}^2 c^4$$

We see that the K$^-$ is highly relativistic(v~c) and therefore minimum ionizing.

* NUMBER OF BUBBLES/CENTIMETRE ~ $1/v^2$.

Now let us return to our massive ($m>>m_e$) charged particle travelling through matter, and try to get a feel for why the rate at which it loses energy (number of bubbles per centimetre, or, to use the jargon of the particle physicist, `ionization density', dE/dx) is inversely proportional to v^2.

At any point along its trajectory, the force exerted on the electron by the projectile can be resolved into components parallel and perpendicular to the direction of motion of the projectile. For every point on the trajectory to the left of the point of closest approach there is a corresponding point to the right, for which the perpendicular component of force on the electron is the same, but the parallel component is in the opposite direction. So to work out what happens to the electron, we need only consider the component of force at right angles to the projectile's motion.

A simple first approach is to use Newton's second law: the time-integral of a force (its impulse I) equals the change in momentum. If the force acts for a short time we can say

$$I \sim \text{average force} \times \text{its duration}$$

To estimate the impulse imparted to the electron by the massive projectile of charge Z and distance of closest approach b, we picture it as feeling a force $Ze \cdot e / 4\pi\varepsilon_0 b^2$ (an overestimate because this is the maximum force) for a time $2b/v$ (an underestimate, because this is the time for which the force is between $0.5F_{max}$ and F_{max}).

We then have an expression for the momentum p imparted to the electron:

$$p = (Ze \cdot e / 4\pi\varepsilon_0 b^2) \times 2b / v \sim 1/v$$

Hence, the kinetic energy picked up $(p^2/2m) \sim 1/v^2$, as required.

(For a detailed derivation, click here. This topic was first discussed in visionary papers by Niels Bohr: *Phil. Mag*. 25, 10 (1913) and *Phil. Mag*. 30, 581 (1915).)

The Physical Principles of Particle Detectors

By Goronwy Tudor Jones

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Most of what is known about particle (and nuclear) physics comes from measuring what emerges from a collision of one particle with another in an instrument called a detector. This resource article is concerned with the basic physics ideas upon which many detectors depend. It includes a careful derivation of the rate at which a heavy charged particle loses energy as it travels through matter.

What is Particle Physics?

The aim of particle physics is to study the fundamental building blocks of nature and the forces they exert on each other. The experimental side of this subject consists of examining what happens when particles are made to collide at high energies at accelerator centers such as CERN (The European Center for Particle Physics Research) in Geneva or Fermilab just outside Chicago.

- **Why do particle physicists need large high-energy accelerators?**

  Surprisingly enough, it is because they are trying to look at something very small, like a quark inside a proton. Let us begin with a basic rule of microscopy—you cannot see anything smaller than about the wavelength of the radiation you are shining on it. One way to come to terms with this idea is by means of the following analogy: you cannot sense the details of braille with a blackboard eraser, but you can with a pencil point.

  Now wave-particle duality tells us that a beam of particles of momentum p corresponds to a wave of wavelength $\lambda = h/p$, where h is Planck's constant. So, to get a very small wavelength one needs a very high momentum; hence the need for accelerators. (This also explains why particle physics is often referred to as high-energy physics.)

  - Quantitative examples include going from red light to violet with photon energies ranging from about 1.5 eV to about 3 eV and corresponding wavelengths going from 8 to 4×10^{-7} meters; this means that it is not possible to see atoms (typically a few times 10^{-10} meters across) with visible light.
  - An x-ray photon with an energy of 25 keV (the dental x-ray region) has a wavelength of 0.5×10^{-10} meters. The same wavelength for an electron can be obtained in an electron microscope in which an electron is accelerated through 0.61 keV. (Lens design for electron beams prohibits the attainment of this calculated resolution.)
  - In particle physics, a typical beam energy of 25 GeV (25×10^9 eV) (of any kind of particle) corresponds to a wavelength of 5×10^{-17} meters, much less than the radius of the proton ($\sim 10^{-15}$ meters).

When one high-energy particle strikes another, it often happens that many particles are produced. It is not, as one might reasonably guess, just a question of the two particles breaking up into many smaller particles. Provided there is enough energy in the initial state, it is possible, for example, to start with two protons and end up with two protons plus several other particles with nonzero rest mass.

For us to make this statement about new particles being created, it is necessary to be able to measure particle masses. Moreover, as far as the particle physicist is concerned, a knowledge of particle masses is crucial because one of the most important ways of distinguishing one particle from another is by measuring its mass. For example, a positive particle with a mass of 1.673×10^{-27} kg or 938.3 MeV/c^2 is a proton.

This article takes the first step along the road to measuring particle masses. Derivations will be made within the framework of classical physics. Asides on how relativity and quantum mechanics modify things will be made where appropriate. Fortunately, the most important results of the classical approach survive.

Determining the Mass of a Particle

The mass, m, of a moving particle can be obtained from a knowledge of two of the following three quantities: momentum, p, speed, v, and kinetic energy, E. These are related by the following equations: $p = mv$; $E = mv^2/2$; and $E = p^2/2m$. These can be rewritten to give three different expressions for mass: $m = p/v$; $m = 2E/v^2$; and $m = p^2/2E$.

♦[Aside: This fact, that the mass can be obtained from a knowledge of any two of the quantities p, v, and E, is also true for highly energetic particles moving with speeds comparable with that of light. The three formulas have to be replaced by their relativistic counterparts:

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \qquad (1)$$

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \qquad (2)$$

$$E^2 = p^2 c^2 + m_o^2 c^4 \qquad (3)$$

In these formulas, the quantity E is the total mass of the particle; m_o is known as its rest mass, the mass that characterizes the particle. (It is the mass that we would use if it were traveling at a nonrelativistic speed with respect to us.)

It is not the aim of this article to introduce or discuss relativistic kinematics. However, it would seem appropriate to point out that a particle is considered to be relativistic if its kinetic energy, E_K, is much greater than its rest energy, $m_o c^2$. (These quantities are related by $E = E_K + m_o c^2$.) When this condition applies, $p^2c^2 >> m_o^2 c^4$ and (3) simplifies to $E = E_K = pc$.

An electron in a TV tube, with a typical energy ~10 keV, is not relativistic, and has a de Broglie wavelength $\sim 0.12 \times 10^{-10}$ meters.

A particle with kinetic energy equal to its rest energy (0.511 MeV for an electron, 938.3 MeV for a proton) travels at 87 percent of the speed of light.

At Fermilab, protons accelerated to 1 TeV (10^{12} eV) travel within 132 m/s of the speed of light!] ♦

Our purpose, now that we know that a knowledge of any two of p, v, and E for a particle enable its mass to be calculated, is to address ourselves to the question of how these quantities can be measured.

We shall see, for massive charged particles, that three easily measurable (in principle) quantities—radius of curvature in a magnetic field (r), rate of energy loss with distance (dE/dx), and total distance traveled before stopping (R, for range)—provide measurements of p, v, and E respectively.

The Bubble Chamber

If two airplanes with vapor trails behind them were to approach each other, circle around, and then go their separate ways, the fact that they had done so would be apparent for quite a while. A permanent record of the encounter could be obtained by taking a photograph of the vapor trails.

A particle detector is an instrument that can record the passage of particles through it. We shall concentrate on one—the bubble chamber—because it is the best visual aid for beginning to come to terms with interactions between particles.

- The bubble chamber consists of a tank of unstable transparent liquid (superheated liquid hydrogen, for example) in which passing charged particles initiate boiling as a result of the energy they deposit along their trajectories. The superheated liquid is prepared by starting with the liquid held under pressure (about 5 atmospheres for hydrogen at a temperature of about 27 K) and then, just before the beam particles arrive, the pressure is reduced by suddenly expanding the volume by about 1 percent by means of a piston.

  After the particles have passed through the liquid, the bubbles are allowed to expand until they are a few tenths of a millimeter across, big enough to photograph by flash illumination. It is interesting to imagine the time scales involved: the relativistic particles cross the few meters of liquid in a few nanoseconds; the growth time is a million times longer, ~10 ms.

  Once the photographs are taken (more than one view is needed to reconstruct an interaction in three dimensions), the bubbles are collapsed by recompressing the liquid.

  The great advantage of bubble chambers is their ability to pick up details of complicated interactions; by following the trails of bubbles one can see subsequent interactions and decays of the products of the initial interaction.

  Sadly, bubble chambers are almost extinct. They cannot cope with the huge event rates of modern fixed-target experiments; nor can they be used with colliding beams.

Figure 1 is a bubble chamber picture of an interaction between a charged particle entering from the bottom and a stationary proton.[a] The white lines are the bubble trails; these are curved because the liquid is embedded in a powerful

[a]Hydrogen provides a target of protons for study, free from the complications of being bound in nuclei.

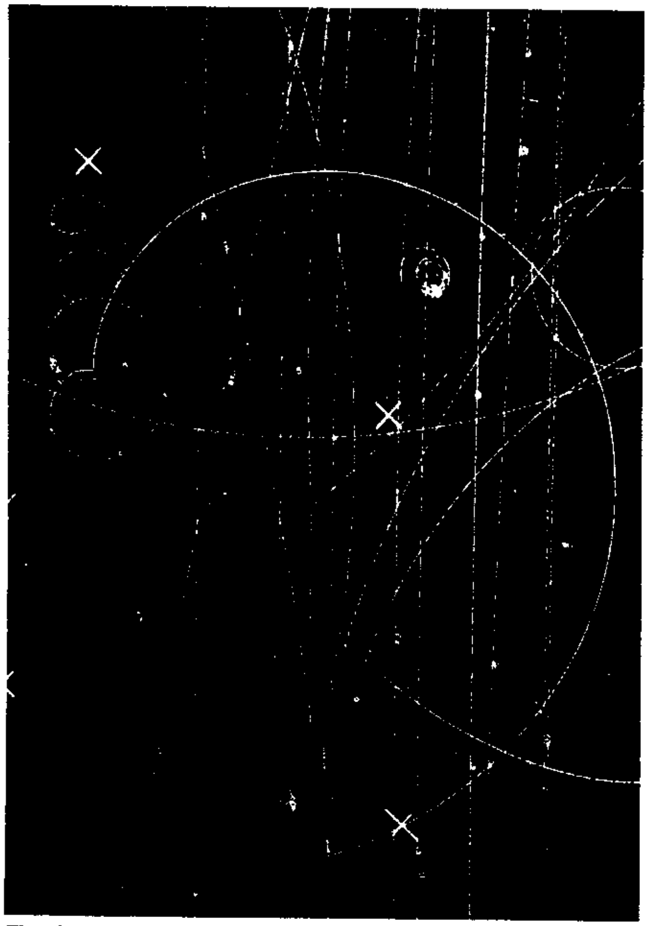

Fig. 1a. Bubble-chamber picture containing an interaction in which one particle strikes another to produce four charged particles and a neutral particle. The white lines are trails of tiny bubbles created as charged particles force their way through a tank of transparent liquid enclosed in a powerful magnet.
The main interaction of interest takes place at A (see Fig. 1b), where an incoming K^- with a momentum of 8.25 GeV/c interacts with a proton to produce a final state with four charged particles. The most curved of these is a positive pion, $\pi+$, which stops and decays at B into a muon, μ^+, and a neutrino, ν, which escapes undetected. The muon then stops and decays at C into an antielectron (or positron), a neutrino, and an antineutrino. (This picture, from an experiment performed in the 2-m hydrogen bubble chamber at CERN, was found at the University of Birmingham.)

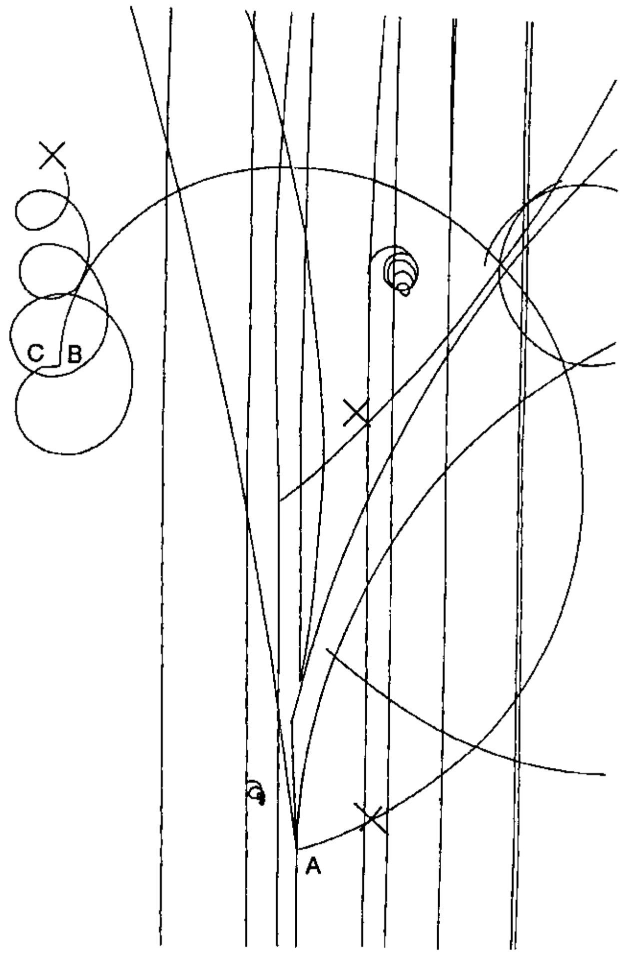

Fig. 1b.

magnetic field, the direction of which is such that negative particles turn to the right. Neutral particles do not leave trails of bubbles because it is the electric force between the moving particle and the electrons of the atoms of the liquid that is responsible for starting bubble growth.

Let us look at the picture more closely. The ten parallel lines coming in from the bottom are projectile particles fired into the bubble chamber in the hope that at least one will interact with a proton—two do! Inspecting the final state particles one observes:

1. Some tracks are more curved than others. We will see later that the more curved the track, the lower the momentum of the particle that made it.

2. Some tracks have more gaps in them than others, due to the fact that there are more bubbles per unit distance. Sometimes it is possible to see that tracks get less sparse towards the end of their paths, suggesting that the slower a particle is moving, the more energy it loses per unit distance. The most curved track from the four-pronged interaction shows a hint of this effect; you can see gaps early on but not near its end.

♦[Aside: It is not essential at this point to know, but this track is produced by a positive pion π^+ which stops and decays into a positive muon μ^+ and a neutrino ν; the muon then stops in a short distance and decays to an antielectron or positron e^+, a neutrino, and an antineutrino $\bar{\nu}$.]♦

3. Some particles stop in the bubble chamber liquid; this happens when all the energy they had has been transferred to the hydrogen atoms of the liquid, initiating boiling. We should not, therefore, be surprised if the range of a particle gave a measure of the kinetic energy it had at the beginning of its trajectory.

Measurement of Momentum

A particle of charge q traveling through a magnetic field B with a speed v experiences a force at right angles to its motion, given by Bqv if the motion is perpendicular to the field. This makes the particle follow a circular path of radius r, the motion being described by

$$Bqv = \frac{mv^2}{r} \tag{4}$$

Rearranging this gives

$$p = (Bq)r \qquad (5)$$

This tells us that for a given field, B, and charge, q, the momentum, p, is proportional to the radius of curvature, r.

Here, nature has been kind: all charged particles that live long enough to travel a measurable distance have a charge equal or opposite to the charge on the electron, e ($= 1.6 \times 10^{-19}$ coulomb).

Equation (5) then becomes, in units used by nuclear and particle physicists: $p = 0.3\ Br$, where p is measured in GeV/c, B in tesla, and r in meters. For the CERN 2-m bubble chamber, in which the interaction in Fig. 1 was photographed, the magnetic field of 1.78 tesla would give a 10 GeV/c track a radius of curvature of about 2 meters. (At the magnetic north pole, the magnetic field at the Earth's surface is 6.2×10^{-5} tesla.)

So, a measurement of the radius of curvature of a track gives its momentum.

♦[Aside: The same formula is true for particles moving with relativistic speeds.]♦

How Does a Charged Particle Lose Energy as It Travels through Matter?

All charged particles, except the electron itself, are at least hundreds of times more massive than the electron. In this section we shall see how an understanding of the way a massive ($M >> m_e$) charged particle loses energy enables us to measure both its speed and kinetic energy.

The force responsible for the energy loss of a charged particle as it travels through matter is the so-called Coulomb force.[b] The energy lost by the traveling particle can raise electrons to higher energy levels (*excitation*) or knock the electrons out of the atoms altogether (*ionization*). When all the kinetic energy of the moving particle is used up, it stops, and so the range R of a particle is a measure of the initial kinetic energy.

To understand this properly, the first step is to be able to calculate the rate at which a particle loses energy with distance, dE/dx. The derivation presented here is a simplified version of the classical (not quantum mechanical or relativistic) argument first given by Bohr in 1913 (see references 2 and 3). It is worth doing because it arrives at the important

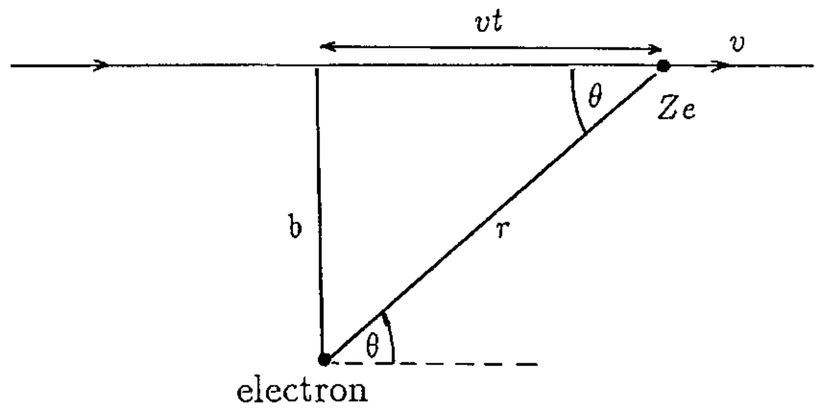

Fig. 2. A massive particle of charge *Ze* and speed *v* passes a stationary electron with impact parameter *b*.

result that dE/dx depends on the speed of the moving particle, but not its mass. Also, the major features survive the full treatment. But, most important, it is one of the great formulas of physics because it is at the heart of most of the detectors ever used in nuclear and particle physics.

Measurement of Speed

The energy lost by a massive charged particle traveling a distance dx is the energy it imparts to one electron times the number of electrons it meets. We will derive the two terms separately, multiply them, and then discuss the assumptions.

Calculation of energy imparted to one electron:

Consider a charged particle of mass M ($>> m_e$, the mass of the electron), charge Ze, and speed v, approaching a stationary electron at distance b from its line of flight (see Fig. 2). Let us simplify the problem by restricting our attention to the case where:

- the projectile is moving quickly, imparting only a little energy or momentum to the electron as it continues along a more or less straight trajectory
- we can ignore the motion of the electron during the interaction, which takes place in a short time when the projectile is near the electron and the electrical force is greatest.

Now, at any point along its trajectory, the force exerted on the electron by the projectile can be resolved into components parallel and perpendicular to the direction of motion of the projectile. For every point on the trajectory to the left of the point of closest approach there is a corresponding point to the right, for which the perpendicular component of force on the electron is the same, but the parallel component is in the opposite direction. So, to work out what happens to the electron, we need only consider the components of force at right angles to the projectile's motion.

A neat way to do this is to use Newton's second law in a form in which it is not often used—the time-integral of a force (its impulse I) equals the change in momentum: $I =$

[b]In 1767, Joseph Priestley, the discoverer of oxygen, was the first to publish the inverse square law of electrostatics. Responding to a letter from Benjamin Franklin, Priestley confirmed that there is no electric field inside a charged conductor, and says in his *The History and Present State of Electricity*[1]: "May we not infer from this experiment, that the attraction of electricity is subject to the same laws with that of gravitation, and is therefore according to the squares of the distances; since it is easily demonstrated, that were the earth in the form of a shell, a body in the inside of it would not be attracted to one side more than another?"

$\int_{t_1}^{t_2} F dt = p_2 - p_1$. This form is particularly useful in situations where a force acts for a short time and we can say that I = average force × its duration.

To estimate the impulse imparted to the electron by the massive projectile, we picture it as feeling a force of about $(Ze)e/4\pi\varepsilon_o b^2$ (an overestimate because this is the maximum force) for a time $2b/v$ (an underestimate, because this is the time for which the force is between 0.5 F_{max} and F_{max}).

The momentum, p, picked up by the electron is thus estimated to be $p = \frac{(Ze)e}{4\pi\varepsilon_o b^2} \times \frac{2b}{v}$.

This estimate turns out to be exact and corresponds to a kinetic energy being imparted to the electron, which is given by

$$E = \frac{p^2}{2m_e} = \frac{1}{16\pi^2\varepsilon_o^2} \times \frac{2(Ze)^2 e^2}{m_e b^2 v^2} \tag{6}$$

We have just calculated the energy imparted to a stationary electron as a heavy projectile passes by, or, equivalently, the energy lost by the projectile to one electron. It is proportional to $1/v^2$ because the momentum imparted to it is proportional to the time the projectile spends close, which $\sim v^{-1}$.

To calculate the energy, dE, lost in a distance, dx, we need to multiply by the number of electrons encountered.

Calculation of the number of electrons met in a distance dx:

We shall do this by calculating the number of electrons at distances between b and $b + db$ from the line of flight. We will then calculate the energy imparted to these, and finally integrate over b.

Let us consider the volume between two concentric cylinders of radii b and $(b + db)$, each having a length dx (see Fig. 3). The number of electrons in the shell will be the volume of the shell × the number, n, of electrons per unit volume: $2\pi b\, db \times dx \times n$.

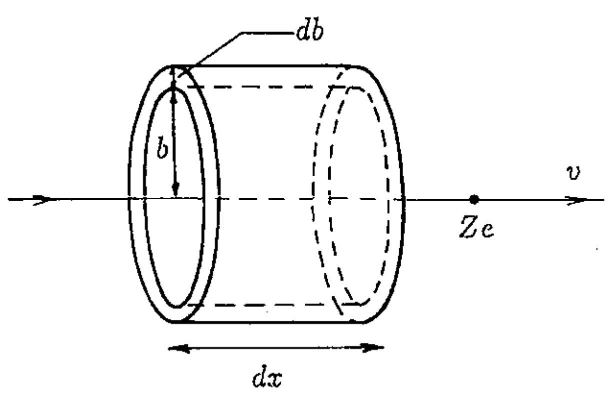

Fig. 3. Two concentric cylinders of radii *b* and *b* + *db* with axis along the direction of motion of the massive particle of charge *Ze* and speed *v*.

Calculation of total energy lost in distance dx:

Using Eq. (6) we see that the energy lost by the projectile to the electrons in the shell at b of length dx is given by

dE (at b) =

$$\frac{1}{16\pi^2\varepsilon_o^2} \times \frac{2(Ze)^2 e^2}{m_e b^2 v^2} \times 2\pi b\, db\, dx\, n \tag{7}$$

Tidying up we get

$$\frac{dE(\text{at } b)}{dx} = \frac{1}{4\pi\varepsilon_o^2} \times \frac{(Ze)^2 e^2 n}{m_e v^2} \times \frac{db}{b} \tag{8}$$

To get the total change in energy in distance dx we must integrate over all impact parameters, b. This gives

$$-\frac{dE}{dx} = \frac{1}{4\pi\varepsilon_o^2} \times \frac{(Ze)^2 e^2 n}{m_e v^2} \times \int_{b_{min}}^{b_{max}} \frac{db}{b} \tag{9}$$

A minus sign has appeared! This is because we have changed from talking about energy loss and are now talking about the change in energy.

Now the number, n, of electrons per unit volume equals the number, Z, of electrons per atom times the number of atoms per gram, N_A/A, times the number of grams per unit volume, ρ. Here, N_A is Avogadro's number and A is the atomic weight of the material.

Performing the integration we get,

$$-\frac{dE}{dx} = \frac{1}{4\pi\varepsilon_0^2} \times \frac{(Ze)^2 e^2}{m_e v^2} \times \frac{N_A Z \rho}{A} \times \ln\frac{b_{max}}{b_{min}} \tag{10}$$

A problem arises! Since the impact parameter b is the perpendicular distance from the electron to the line of flight of the projectile, it would seem natural for b_{min} and b_{max} to be 0 and ∞ respectively. This would, however, lead to an infinite rate of loss of energy dE/dx. Here we shall address the question *Why should b_{min} and b_{max} be nonzero and finite?* The actual values are derived in Appendix A.

Why is b_{min} nonzero? Look at Eq. (6). The b^2 in the denominator implies that the energy imparted to the electron is infinite for $b = 0$. This formula is clearly not valid for a head-on collision with a particle moving with finite speed v. We'll see later that b_{min} depends on v but not on the mass of the heavy ($M >> m_e$) projectile.

Why is b_{max} not infinite? If the electron is bound in an atom, the kinetic energy it receives (which $\sim b^{-2}$) must be enough to enable it to reach a higher energy level. So b_{max} must depend on energy gaps in the atom. Let us introduce a symbol $\bar{I}$ to describe this, where $\bar{I}$ is the average value of ($E_n - E_0$), the energy needed to excite an atom to a level of energy, E_n, from the ground state of energy, E_0.

These qualitative arguments allow us to say that $\ln\frac{b_{min}}{b_{max}} = f(v, \bar{I})$.

♦[Aside: Equations (14) and (19) in Appendix A show that $b_{max}/b_{min} = (2m_e v^2/\bar{I})^{1/2}$. Using this value we will be able to make a numerical calculation of the energy deposited per centimeter of a bubble chamber track.]♦

The expression for dE/dx tells us that for a given material (characterized by the number of electrons per unit volume, n, and the average excitation energy, $\bar{I}$), a particle of charge, Z, loses energy at a rate that depends *only* on its speed.

This means that we can determine the speed, v, of a particle by measuring the rate at which it loses energy (*stopping power* or *ionization density* in the jargon of nuclear and particle physicists).

To the bubble-chamber physicist, ionization density means the number of bubbles per centimeter, something that can be counted to estimate the speed of a massive particle.

Measurement of Energy

We have just shown that $-\frac{dE}{dx} = f(v, \bar{I})$, i.e., $dx = -\frac{dE}{f(v,\bar{I})}$.

We can integrate this equation from the beginning of the trajectory ($x = 0$ and $E = E$) to its end ($x = R$ and $E = 0$):

$$\int_R^0 dx = -\int_E^0 \frac{dE}{f(v,\bar{I})} \tag{11}$$

This is the relationship we need—it shows that measuring the range, R, of a charged particle moving through matter gives its energy.

Before moving on, it is instructional to consider some concrete applications. Using Eq. (10), and changing the variable from $E = Mv^2/2$ to v via $dE = Mv\,dv$, Eq. (11) gives

$$R = \frac{4\pi\varepsilon_o^2 m_e}{e^4 N_A} \times \frac{A}{\rho Z} \times \frac{M}{Z^2} \times \int_0^v \frac{v^3 dv}{\ln\sqrt{\frac{2m_e v^2}{\bar{I}}}} \tag{12}$$

Here R is expressed as the product of four terms: the first consists of constants; the second depends on the medium; the third depends on the projectile; and the last depends only on v (if we ignore any Z-dependence of $\bar{I}$).

Let us consider the case of two particles of the same speed traveling through the same medium: $R \propto M/Z^2$. This tells us that a deuteron, having twice the mass but the same charge as the proton, will travel twice as far as a proton.

If the two particles have the same charge as well, $R \propto M$. We shall now see how this relationship can be used to determine the mass of a muon relative to the mass of the pion from which it emerges as a decay product ($\pi \to \mu + \nu$). In a photographic emulsion, a μ from the decay of a stopped π had a range of 0.6 mm. A scanner noticed that the ionization density (number of blackened grains per unit distance) of the μ at the beginning of its path was the same as that of the parent π at a distance of 0.793 mm from the decay point. This tells us that a π and a μ of the same speed had ranges of 0.793 mm and 0.6 mm respectively; hence the mass of the μ is 0.6/0.793 times that of the π.

Next, let us consider one particle of fixed speed traveling through two different media denoted by X and Y. The ratio of the ranges R_X and R_Y is given by

$$\frac{R_X}{R_Y} = \frac{A_X}{A_Y} \times \frac{\rho_Y Z_Y}{\rho_X Z_X} \tag{13}$$

For light materials (other than hydrogen), $Z \approx A/2$, so $\frac{R_X}{R_Y} \approx \frac{\rho_Y}{\rho_X}$. In other words the range is inversely proportional to the density.

Summary

Three easily measurable (in principle, at least) properties of a track (in a bubble chamber, for example)—curvature, ionization density, and range—provide determinations of the momentum, speed, and kinetic energy of the particle responsible for the track.[c] From any two of these, the mass of this particle can be calculated [redacted]

Appendix A

What are the limits on the impact parameter?

What is the lower limit b_{min}?

Equation (6) is clearly not valid for $b = 0$ because it says that a massive particle of finite energy can lose an infinite amount of energy to an electron in a head-on collision.

The easiest way to see how much energy can be transferred to the electron is to picture the collision in the rest frame of the massive projectile: if the collision is elastic, the electron merely has its velocity reversed by the impact—just like throwing a golf ball at a wall.

Relative to the massive projectile, the light electron will bounce off with a speed, v. In the laboratory system, in which the projectile already has speed v, the electron's speed is $2v$. This corresponds to the electron picking up a kinetic energy of $0.5\ m_e(2v)^2$, which is the most that the projectile can transfer to the electron.

If we are to express the energy transferred to the electron in terms of an impact parameter, it is clearly meaningless to contemplate values for b that correspond to the electron acquiring more kinetic energy than is kinematically allowable. The value for b_{min} is thus the value of b that yields the maximum energy the electron can be given:

$$\frac{1}{2}(2m_e v)^2 = \frac{1}{16\pi^2\varepsilon_o^2} \times \frac{2(Ze)^2 e^2}{m_e b_{\min}^2 v^2}$$

$$b_{\min} = \frac{1}{4\pi\varepsilon_o} \times \frac{(Ze)e}{m_e v^2} \tag{14}$$

♦[Aside: Students have remarked that it helps their appreciation of the argument presented above, in the reference frame of the

[c]Remember that all this has been shown for particles much heavier than the electron. The way electrons lose energy as they travel through matter is almost another subject! They are instantly recognizable in a bubble chamber because their tracks spiral.

projectile, to see it done by straightforwardly applying momentum and energy conservation in the laboratory frame.

Let the final speeds of M and m_e be W and w; then, by momentum conservation $Mv = MW + m_e w$, i.e., $v = (W + \frac{m_e}{M} w)$.

For maximum energy transfer to m, the collision must be elastic; so, by energy conservation

$$\frac{1}{2}Mv^2 = \frac{1}{2}MW^2 + \frac{1}{2}m_e w^2 \tag{15}$$

Substituting for v gives

$$\frac{1}{2}M(W^2 + \frac{2m_e}{M}Ww + \frac{m_e^2}{M^2}w^2) = \frac{1}{2}MW^2 + \frac{1}{2}m_e w^2 \tag{16}$$

For $m_e << M$ we ignore the term with $(m_e/M)^2$ to get $m_e Ww \approx \frac{1}{2}m_e w^2$ or $w \approx 2W$.

The light particle emerges from the collision with about twice the speed of the heavy one. We shall now see that the speed of the heavy one is little changed (as one might expect, for example, if a cannon ball hit a marble). Substituting $w = 2W$ into the momentum conservation equation gives

$$Mv = MW + m_e w = MW + 2m_e W \tag{17}$$

which implies that $\frac{W}{v} = \frac{M}{M + 2m_e}$. In other words, the speed of the projectile is reduced very little by its collision with the light *electron*, which, in turn moves with a speed of nearly $2v$.]♦

What is the upper limit b_{max}?

An electron in an atom cannot absorb an arbitrarily small amount of energy; it must receive enough to raise it to a higher energy level. We have already introduced a quantity, $\bar{I}$, which can be taken as a typical minimum energy the electron must receive. Equating this to the energy imparted to an electron [Eq. (6)] we get

$$\bar{I} = \frac{1}{16\pi^2\varepsilon_o^2} \times \frac{2(Ze)^2e^2}{m_e b_{\max}^2 v^2} \tag{18}$$

i.e.,

$$b_{\max} = \frac{Ze^2}{4\pi\varepsilon_o v}\sqrt{\frac{2}{m_e \bar{I}}} \tag{19}$$

♦[Aside: In Bohr's original paper (submitted in 1912, before the Bohr Model of the atom, let alone quantum mechanics!) there is a beautiful classical discussion of why the energy transfer to an electron is negligible if the duration of the impulse is more than the *vibration period*, $1/f$, of the electron in the atom. Here we present a short quotation, modified slightly to match this text:

"If the time of collision is very short compared with the time of vibration of the electron, the interatomic forces will not have time to act before the particle has escaped again from the atom, and the energy transferred to the electron will therefore be very nearly the same as if the electron were free. If, on the other hand, the time of collision is long compared with the time of vibration, the electron will behave almost as if it were rigidly bound, and the energy transferred will be exceedingly small. The effect of the interatomic forces is therefore equivalent to the introduction of an upper limit for b in the integral in (7), of the same order of magnitude as v/f."

A nonrelativistic 10-MeV (1.6×10^{-12}J) proton has a speed ~4.4 $\times 10^7$ m/s that's considerably faster than the electron in the first Bohr orbit—2.2×10^6 m/s. Such a proton is within an atomic diameter for much less than one period of the electron.

A more accessible reference to Bohr's estimate of b_{max} is the textbook by Fermi.[4] That book also contains a discussion of how quantum mechanics puts a limitation on the validity of the argument presented above for estimating b_{min}.]♦

Now that we have explicit expressions for b_{min} and b_{max}, let us check a rule-of-thumb of bubble chamber physicists: a highly relativistic particle loses about 0.23 MeV/cm in liquid hydrogen (density = 63 kg/m^3).

Equations (14) and (19) give $b_{max}/b_{min} = (2m_e v^2/\bar{I})^{1/2}$. For a relativistic projectile $v \sim c$, let us take $\bar{I} \approx 13.6$ eV and remember that $m_e = 0.511$ MeV/c^2. Then $b_{max}/b_{min} \approx 274$ and $\ln(b_{max}/b_{min}) \approx 5.6$.

The equation we now have to put numbers into is:

$$-\frac{dE}{dx} = \frac{1}{4\pi\varepsilon_o^2} \times \frac{(Ze)^2e^2}{m_e v^2} \times \frac{N_A Z\rho}{A} \times \ln\frac{b_{\max}}{b_{\min}} \tag{20}$$

If we use

$1/4\pi\varepsilon_o = 8.987 \times 10^9$ Nm^2/C^2	$e = 1.6 \times 10^{-19}$ C
1 J = 6.24×10^{18} eV	$m_e = 9.1 \times 10^{-31}$ kg
$N_A = 6 \times 10^{23}$ atoms per mole	$A = 2$
$\rho \approx 63$ kg per cubic meter	$Z = 1$

we end up with

$-dE/dx \approx 0.11$ MeV/cm.

This is encouragingly close to 0.23 MeV/cm. The extra factor of 2 is obtained if we use the more exact result (which uses the full armory of quantum mechanics and relativity) known as the Bethe-Bloch equation:

$$-\frac{dE}{dx} = \frac{1}{4\pi\varepsilon_o^2} \times \frac{(Ze)^2e^2}{m_e v^2} \times \frac{N_A Z\rho}{A} \times \ln\frac{2m_e v^2}{\bar{I}(1-\beta^2)} - \beta^2 \tag{21}$$

where $\beta = v/c$. Notice that the log term now has no square root (and dE/dx is roughly twice the value we have calculated); a good but difficult treatment can be found in Bethe's *Intermediate Quantum Mechanics*.[5]

Because lead (for example) is more dense than hydrogen, there are more electrons per unit distance in it. Consequently, dE/dx is greater for lead. If we divide dE/dx by the density of the material, we get a quantity $dE/d(\rho x)$, known as the specific energy loss, which is roughly the same for all materials.

Figure 4 shows how $dE/d(\rho x)$ for a proton projectile varies with kinetic energy for hydrogen and lead. Notice that there is a minimum about 1 GeV (which is when the kinetic energy is comparable with the rest energy, $m_p c^2$, of the proton projectile). Below this energy the curve is dominated by the $1/v^2$ term. The rise from minimum ionization levels off at about 1.3 times the minimum value and is due to relativistic effects (see Ref. 4). (In passing, the fact that the ionization density varies so little once the particles reach a kinetic

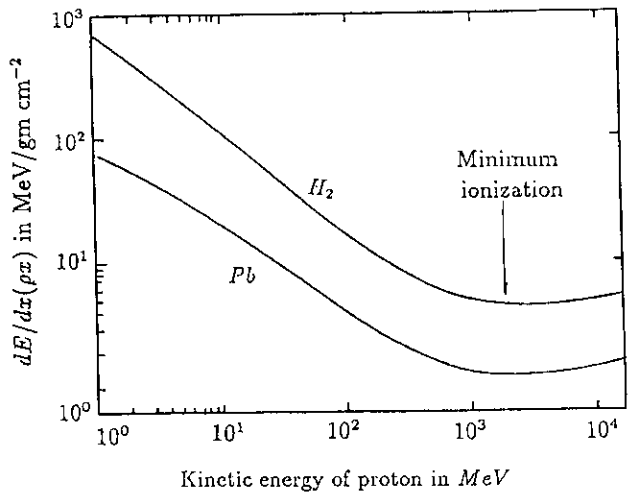

Fig. 4. Specific energy loss $dE/d(\rho x)$ depends on the kinetic energy of a proton traveling through hydrogen and lead.

energy close to their rest mass means that it is not easy to use this quantity to measure speeds close to that of light. A detector called a Čerenkov detector is often used to get information on the speed of a highly relativistic particle—but this is not the place to go into other detectors!)

It is worth noting that for all materials $dE/d(\rho x)$ is about 2 MeV/gm cm^{-2} at minimum. Finally, remember that the Bethe-Bloch equation is not valid for projectile speeds comparable to the speed of an electron within an atom—positive particles can pick up electrons!

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