## BUBBLE CHAMBER ANALYSIS

## SECTION I - PRODUCING A BUBBLE CHAMBER PHOTOGRAPH

The thirteen slides accompanying the apparatus are photographs of tracks made by a beam of negative pi-mesons (pions) as they traverse a bubble chamber filled with liquid hydrogen. The following is a brief description of the methods used to obtain these photographs.

Elementary particles are produced by high energy collisions. Such collisions occur when a target of nuclear matter is bombarded by particles travelling at high speeds - they often occur naturally in stars, giving rise to the well-known cosmic rays. Thus, to create elementary particles by breaking apart nuclear matter, it is necessary to produce a beam of particles (usually protons) with which to bombard a target.

The pictures for this experiment were taken at Brookhaven National Laboratory in New York. The bombarding particles were produced at the Alternating Gradient Synchrotron (AGS). The AGS is a doughnut shaped machine with a diameter of about 850 feet, which accelerates protons to speeds approaching that of light - to energies as high as 30 billion (Giga) electronvolts (eV), and with an intensity of $10^{12}$ to $10^{13}$ protons per machine pulse. A detailed description of how an accelerator like the AGS operates is available elsewhere and we shall not dwell further on it here.

After the proton beam has been accelerated to a sufficient energy and intensity, the target, typically made of aluminum, tungsten, or some other high $Z$ element, is inserted into the path of the beam. When the bombarding protons strike the target many different elementary particles are produced. The next step then consists of selecting from this array particles of a given mass at a well-defined energy. This is done by placing electric and magnetic fields in the vicinity of the target. The purpose of these fields is to select from the different particles travelling at many different energies, all those particles which have a particular mass and which are travelling with a particular energy. Having selected a particular energy and being satisfied that the beam is a pure sample of a given mass (beam purity is typically $>85 \%$ ) the physicist is ready to perform a controlled experiment. To observe interactions between this selected beam (negative pions in our case) and matter, detectors such as cloud chambers, spark chambers, counters, or bubble chambers are employed. The particular type of detector used depends upon the requirements of the experiment.

The detector used in the present case was a bubble chamber. The bubble chamber is filled with liquid hydrogen (the chamber can be filled with other liquids - helium, for example, is often used) and is kept at a temperature near 27 K and a pressure of about 48 psi . At this temperature and pressure liquid hydrogen can become super-heated. A liquid is said to be super-heated when, at a given pressure, it remains in the liquid state even though its temperature exceeds the boiling point. When a liquid is in a super-heated state a very small disturbance can cause it to start boiling spontaneously. As a charged particle travels through the super-heated liquid hydrogen it can transfer some of its energy to the electrons in the liquid, causing these electrons to gain kinetic energy. The increased microscopic motion of the electrons is a sufficient disturbance to cause localized boiling along the path of the charged particle. The bubbles which result are then photographed.

The bubble chamber is placed in a magnetic field. The camera locations and direction of the magnetic field can be seen from Figures 1 and 2. All dimensions are approximate.

As is evident from Figure 2, the beam tracks, which are made by negative pions, will curve downward. This curving is due to the magnetic force (the Lorentz force, $\mathbf{F}=q \mathbf{v} \times \mathbf{B}$ ) which acts on a charged particle when it travels through a magnetic field. The beam tracks, initially in a plane parallel to the front glass, stay in a plane parallel to the front glass because the magnetic field is perpendicular to the glass. After a picture is taken, the piston (shown in Figures 1 and 2) compresses the hydrogen, thereby raising its boiling point so that it returns to a pure liquid state. Hence the bubbles disappear. The piston is then withdrawn, lowering the boiling point, so that the hydrogen is returned to a super-heated state. Once the chamber has been restored to the proper operating condition the beam enters again, bubbles are formed, the light source flashes, and a picture is taken. The rate of the expansion-compression cycle is coordinated with the rate at which the AGS pulses, and hence with the rate at which the pions reach the chamber. The repetition time for this cycle is about 2.8 seconds. The film is then developed and the pictures can be examined and analyzed.

In summary, the process by which a bubble chamber photograph is obtained consists of five essential steps:

1. An accelerator beam of satisfactory energy and intensity is obtained.
2. A target is inserted into this beam and many elementary particles are created with many different energies.
3. Particles of a particular mass and energy are selected to form a secondary beam.
4. This secondary beam is directed into the bubble chamber.
5. Pictures are taken of the interactions which occur between this secondary beam and the particles comprising the liquid in the chamber.


Figure 1. Chamber as seen from the beam direction


## SECTION II - RELATIVISTIC KINEMATICS

High energy physics concerns itself with particles of small mass that travel very close to the speed of light. The equations which are used to describe the motion of particles travelling at these speeds need modification as compared to the equations that one used when dealing with more massive objects like bullets or rockets, which typically travel at much slower speeds. These modifications are the result of Einstein's Special Theory of Relativity. Whether one describes relativistic or non-relativistic motion, nature still requires the conservation of energy, linear momentum, angular momentum, and other quantities such as charge and baryon number. In this section we will concern ourselves only with the conservation of energy and linear momentum.

Non-relativistically, an inelastic collision is one in which kinetic energy is not conserved, but linear momentum is conserved. An elastic collision is one in which both kinetic energy and linear momentum are conserved. Relativistic collisions, both elastic and inelastic, conserve linear momentum as well as a quantity called the total energy. The total energy of a particle is the sum of its rest mass energy and the energy it has by virtue of its speed (its relativistic kinetic energy). If a particle of mass $M$ is at rest, its total energy $E$ is its rest energy, or self energy, which is given by the famous Einstein formula

$$
\begin{equation*}
E=M c^{2} \tag{1}
\end{equation*}
$$

where $c$ is the speed of light. Now suppose the particle is not at rest. Let $T$ denote the relativistic kinetic energy (which is not simply $1 / 2 M v^{2}$ ). The total energy of the particle is given by

$$
\begin{equation*}
E=M c^{2}+T \tag{2}
\end{equation*}
$$

As explained previously, the bubble chamber is placed in a magnetic field. Hence, all charged particles travelling through the bubble chamber experience a force which causes them to travel in circular paths. The magnitude of this force is $q \mathrm{VB}$, where $q$ is the electric charge of the particle, V is its speed, and $B$ is the strength of the magnetic field. Since the magnetic force acts perpendicularly to the particle velocity, the particles move in circular trajectories. The centripetal force is equal to the magnetic force:

$$
\begin{equation*}
\frac{M \mathrm{v}^{2}}{R}=q \mathrm{VB} \tag{3}
\end{equation*}
$$

where $R$ is the radius of curvature of the particle's path. From equation (3), since $M \mathbf{v}=\mathbf{p}$ (the particle's momentum) it follows that:

$$
\begin{equation*}
p=q B R \tag{4}
\end{equation*}
$$

In bubble chamber experiments, the measured quantity is $R$, and thus the momentum can be found from equation (4). Therefore, we shall use, instead of equation (2), the following expression which relates the total energy and the momentum:

$$
\begin{equation*}
E=\sqrt{p^{2} c^{2}+M^{2} c^{4}} \tag{5}
\end{equation*}
$$

Note from Equation (5) that $p c$ and $M c^{2}$ must have units of energy. A calculation of $M c^{2}$ for an electron at rest yields:

$$
M c^{2}=\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)^{2}=8.199 \times 10^{-14} \mathrm{~J}=0.511 \mathrm{MeV}
$$

Isolating $M$, but leaving $c^{2}$ as a symbol, rather than substituting its value, yields:

$$
M=0.511 \mathrm{MeV} / \mathrm{c}^{2}
$$

Thus the $\mathrm{MeV} / \mathrm{c}^{2}$ is a valid unit of mass. Similarly it can be shown that the $\mathrm{MeV} / \mathrm{c}$ is a valid unit of momentum.

Note that when a mass expressed in $\mathrm{MeV} / \mathrm{c}^{2}$ and a momentum expressed in $\mathrm{MeV} / \mathrm{c}$ are substituted into equation (5), the value of $E$ can be calculated without ever actually substituting for $c$, the speed of light, since the $c$ symbols in the units and equation cancel. Because of this cancellation, physicists often become careless and write equation (5) as:

$$
\begin{equation*}
E=\sqrt{p^{2}+M^{2}} \tag{6}
\end{equation*}
$$

where it is understood that the momentum value is to be expressed in units of [energy]/c, the mass value is to be expressed in units of [energy] $/ \mathrm{c}^{2}$, and no substitution is to be made for the value of $c$.

Consider dimensional analysis of equation (4) for a singly-charged particle,

$$
[p]=[q][B][R]=[\mathrm{e}]\left[\mathrm{V} \cdot \mathrm{~s} / \mathrm{m}^{2}\right][\mathrm{m}]=\mathrm{eV} \cdot \mathrm{~s} / \mathrm{m}
$$

That is, if $B$ and $R$ are expressed in SI Units (Tesla and metres, respectively), then the momentum will have units of $\mathrm{eV} \cdot \mathrm{s} / \mathrm{m}$.

Multiplying top and bottom of equation (4) by $c$ :

$$
p=B R\left(\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{\mathrm{c}}\right)=300 B R \mathrm{MeV} / \mathrm{c} \text { for } B \text { in } \mathrm{T} \text { and } R \text { in } \mathrm{m} .
$$

Other unit combinations for $B$ and $R$ yield:

$$
\begin{array}{ll}
p=30 B R \mathrm{MeV} / \mathrm{c} & \text { for } B \text { in } \mathrm{kG} \text { and } R \text { in } \mathrm{m} \\
p=0.30 B R \mathrm{MeV} / \mathrm{c} &  \tag{8}\\
\text { for } B \text { in } \mathrm{kG} \text { and } R \text { in } \mathrm{cm}
\end{array}
$$

Before developing the kinematic equations needed for this experiment we must define elastic and inelastic processes from the point of view of elementary particle physics. An elastic collision
is one in which the particles that appear after the collision are identical to those present before the collision, i.e.

$$
\begin{equation*}
\mathrm{A}+\mathrm{B} \rightarrow \mathrm{~A}+\mathrm{B} \tag{9}
\end{equation*}
$$

where in our case particle " A " is a negative pion and " B " is a proton.
An inelastic process is one in which the particles that appear after the collision are not the same as the particles present before the collision. In the inelastic interaction, more or different particles are created - particles which were not necessarily present before the interaction took place. This can occur because mass and energy are interchangeable. One can picture the interaction, when particles "A" and "B" collide, as forming a unique quantity of energy, which then reappears in the form of particles which are not necessarily the same as "A" and "B", and which have varying amounts of momentum. For this experiment if we consider an inelastic interaction,

$$
\begin{equation*}
\mathrm{A}+\mathrm{B} \rightarrow \mathrm{C}+\mathrm{D}+\mathrm{E} \tag{10}
\end{equation*}
$$

then those particles which have been found to appear most frequently on the right hand side of equation (10) are:

$$
\begin{equation*}
\pi^{-}+\mathrm{p} \rightarrow \pi^{-}+\mathrm{p}+\pi^{0} \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
\pi^{-}+\mathrm{p} \rightarrow \pi^{-}+\pi^{+}+\mathrm{n} \tag{12}
\end{equation*}
$$

We shall now discuss the relativistic kinematics for an elastic collision. From equation (9) we have

$$
\begin{equation*}
\pi^{-}+\mathrm{p} \rightarrow \pi^{-}+\mathrm{p} \tag{13}
\end{equation*}
$$

We can schematically represent an elastic event with a diagram like that shown in Figure 3, where the arrows represent the magnitudes and directions of the momenta of the particles. For the purpose of mathematical development we have called the outgoing pion " C " and the outgoing proton "D".

Figure 3


Conservation of energy requires that

$$
\begin{equation*}
E_{\mathrm{A}}+E_{\mathrm{B}}=E_{\mathrm{C}}+E_{\mathrm{D}} \tag{14}
\end{equation*}
$$

where $E$ is defined by equation (6). The target particle, the proton, is at rest, so equation (14) becomes, using equation (6):

$$
\begin{equation*}
\sqrt{p_{\mathrm{A}}^{2}+M_{\mathrm{A}}^{2}}+M_{\mathrm{B}}=\sqrt{{p_{\mathrm{C}}^{2}+M_{\mathrm{C}}^{2}}^{2}}+\sqrt{{p_{\mathrm{D}}^{2}}^{2}+M_{\mathrm{D}}^{2}} \tag{15}
\end{equation*}
$$

$p_{\mathrm{C}}$ and $p_{\mathrm{D}}$ are directly measurable quantities. $M_{\mathrm{B}}$ is the mass of the proton and $p_{\mathrm{A}}$ is a constant $=$ $919 \mathrm{MeV} / \mathrm{c}$.

Hence, equation (15) can be verified directly.
Linear momentum conservation is expressed by the vector equation

$$
\begin{equation*}
\vec{p}_{\mathrm{A}}=\vec{p}_{\mathrm{C}}+\vec{p}_{\mathrm{D}} \tag{16}
\end{equation*}
$$

To determine whether or not momentum is conserved we must check both the $x$ and $y$ components. Choosing the incoming pion in the $x$-direction, and having measured $\theta_{\mathrm{C}}$ and $\theta_{\mathrm{D}}$ directly, equation (16) becomes two scalar equations.

For the $x$-direction we have

$$
\begin{equation*}
p_{\mathrm{A}}=p_{\mathrm{C}} \cos \theta_{\mathrm{C}}+p_{\mathrm{D}} \cos \theta_{\mathrm{D}} \tag{17}
\end{equation*}
$$

and for the $y$-direction

$$
\begin{equation*}
0=p_{\mathrm{C}} \sin \theta_{\mathrm{C}}+p_{\mathrm{D}} \sin \theta_{\mathrm{D}} \tag{18}
\end{equation*}
$$

We now consider the inelastic hypothesis

$$
\begin{equation*}
\mathrm{A}+\mathrm{B} \rightarrow \mathrm{C}+\mathrm{D}+\mathrm{E} \tag{19}
\end{equation*}
$$

In the chamber such an event may look the same as an elastic event. Figure 4 is a schematic representation of an inelastic event.

## Figure 4



In this case, one of the particles, say " E ", is not visible in the chamber since it has no electric charge. However, from energy and momentum conservation not only can we detect the presence of "E", we can determine its mass and momentum. Again, the measured quantities will be $p_{\mathrm{C}}$, $p_{\mathrm{D}}, \theta_{\mathrm{C}}$, and $\theta_{\mathrm{D}} . p_{\mathrm{A}}=919 \mathrm{MeV} / \mathrm{c}$ and $p_{\mathrm{B}}=0$ since the target proton is assumed to be at rest. Conservation of linear momentum gives:

$$
\begin{equation*}
\vec{p}_{\mathrm{A}}=\vec{p}_{\mathrm{C}}+\vec{p}_{\mathrm{D}}+\vec{p}_{\mathrm{E}} \tag{20}
\end{equation*}
$$

Considering the $x$-axis to be in the direction of the incident particle, equation (20) becomes, for the $x$-direction,

$$
\begin{equation*}
p_{\mathrm{A}}=p_{\mathrm{C}} \cos \theta_{\mathrm{C}}+p_{\mathrm{D}} \cos \theta_{\mathrm{D}}+p_{\mathrm{E}} \cos \theta_{\mathrm{E}} \tag{21}
\end{equation*}
$$

and for the $y$-direction,

$$
\begin{equation*}
0=p_{\mathrm{C}} \sin \theta_{\mathrm{C}}+p_{\mathrm{D}} \sin \theta_{\mathrm{D}}+p_{\mathrm{E}} \sin \theta_{\mathrm{E}} \tag{22}
\end{equation*}
$$

We may rewrite equations (21) and (22) using subscripts to specify the $x$ and $y$ directions,

$$
\begin{align*}
& p_{\mathrm{Ax}}=p_{\mathrm{Cx}}+p_{\mathrm{Dx}}+p_{\mathrm{Ex}}  \tag{23}\\
& 0=p_{\mathrm{Cy}}+p_{\mathrm{Dy}}+p_{\mathrm{Ey}} \tag{24}
\end{align*}
$$

Since particle "E" is not visible, $p_{\mathrm{E}}$ and $\theta_{\mathrm{E}}$ are not directly measurable quantities. Hence, for inelastic events, one will find that

$$
\vec{p}_{\mathrm{A}} \neq \vec{p}_{\mathrm{C}}+\vec{p}_{\mathrm{D}}
$$

In other words, for an inelastic event, momentum will not be conserved by the visible, charged particles. When this is found to be the case, one can hypothesize the presence of a neutral particle and can construct its momentum vector so that momentum will be conserved,

$$
\begin{equation*}
p_{E}=\sqrt{p_{\mathrm{Ex}}^{2}+p_{\mathrm{Ey}}^{2}}=\sqrt{\left(p_{\mathrm{Ax}}-p_{\mathrm{Cx}}-p_{\mathrm{Dx}}\right)^{2}+\left(p_{\mathrm{Cy}}+p_{\mathrm{Dy}}\right)^{2}} \tag{25}
\end{equation*}
$$

The direction of " E " is given by

$$
\begin{equation*}
\theta_{\mathrm{E}}=\arctan \left(\frac{p_{\mathrm{Ey}}}{p_{\mathrm{Ex}}}\right)=\arctan \left[\frac{-\left(p_{\mathrm{Cy}}+p_{\mathrm{Dy}}\right)}{\left(p_{\mathrm{Ax}}-p_{\mathrm{Cx}}-p_{\mathrm{Dx}}\right)}\right] \tag{26}
\end{equation*}
$$

More conclusive proof of the presence of particle " E " is provided by the requirement that energy also be conserved. If there is no neutral particle produced after the collision then

$$
E_{\mathrm{A}}+E_{\mathrm{B}}=E_{\mathrm{C}}+E_{\mathrm{D}}
$$

If this equality does not hold, then

$$
\begin{equation*}
E_{\mathrm{A}}+E_{\mathrm{B}}=E_{\mathrm{C}}+E_{\mathrm{D}}+E_{\mathrm{E}} \tag{27}
\end{equation*}
$$

In this experiment, conservation of linear momentum and conservation of total energy will be used to analyze the observed collisions and decide which interaction (equation (11), (12), or (13)) occurred.

If linear momentum and total energy are conserved for the visible particles, then the interaction was elastic.

If linear momentum is not conserved for the visible particles, the neutral particle momentum is obtained from equation (25). Using this momentum and the accepted mass for the neutral particle being considered, a check of conservation of total energy is made by using equation (27). It should be noted that the outgoing positive-charged particle is a proton in interactions (11) and (13), and a positive pion in interaction (12).

The interaction that "best" conserves total energy within experimental uncertainty is chosen as the one that occurred. (Note that we are requiring that linear momentum be conserved in order to determine the neutral particle momentum.)

If it is decided that an interaction was inelastic, an experimental value for the mass of the neutral particle can be obtained by solving equation (27) for $E_{\mathrm{E}}$, and using equation (6).

$$
E_{\mathrm{E}}=E_{\mathrm{A}}+E_{\mathrm{B}}-E_{\mathrm{C}}-E_{\mathrm{D}}
$$

and

$$
\begin{equation*}
M_{\mathrm{E}}=\sqrt{E_{\mathrm{E}}^{2}-p_{\mathrm{E}}^{2}} \tag{28}
\end{equation*}
$$

## SECTION III - TECHNIQUES AND DETERMINATION OF EXPERIMENTAL CONSTANTS

Stereoscopic Reconstruction of Paths in the Bubble Chamber
The coordinate system in the chamber is usually defined so that the $y$ and $z$ axes are perpendicular to the beam direction, as shown in Figure 5.

Figure 5


It was mentioned in Section II that there are four cameras available with which to photograph the interior of the bubble chamber. Why is it necessary to have more than one camera? When an interaction takes place in the chamber, the outgoing tracks may follow any path; they need not travel in planes parallel to the $x$ - $y$ plane. They can, and usually do, travel toward either the front or back glass of the chamber. In order to determine how much a track "dips" toward either glass it is necessary to obtain depth perception in the chamber. For example, a track which heads directly toward a given camera will look like a point to that camera, whereas another camera will see some of the track's "length".

Just as one needs both eyes to enable the brain to "calculate" distances, the experimenter needs at least two "eyes" (cameras) to determine depth in the chamber. The determination of the particle paths in space is called "stereoscopic reconstruction". In principle such a reconstruction is not difficult, although the algebraic calculations are long and tedious. Particle physics labs have developed sophisticated computer programs to perform these calculations.

This experiment, however, does not require stereoscopic reconstruction. The reason for this is that all the events have been chosen so that all tracks lie in planes parallel to the $x-y$ plane - in other words, there are no dipping tracks. That is why, for this experiment, only one camera view is used. (Actually, events were accepted provided that no track dipped more than a few degrees from the $x$ - $y$ plane.)

## Purpose of the Experiment

The purpose of this experiment is to search the slides for interactions between the incoming negative pions and the target protons in the chamber, and then, using the relativistic kinematics outlined in Section II, to determine what type of interaction has occurred.

In order to use the relativistic equations, we must measure the radii of curvature and find the directions of all the visible particles involved in the interaction.

## Method for Measuring Radii of Curvature and Angles

Radii of Curvature

The projection unit should be mounted near the top of the chrome pole, just under the curtain rod clamp, so that as large an image as possible is projected onto the table. Once the projection unit has been adjusted to a suitable position it should not be moved for the remainder of the experiment.

Place a slide in the slide holder with the slide number facing you and at the top, as shown in Figure 6. Insert the slide holder into the projection slit. Adjust the focus of the projection unit to obtain a sharp image on the table.


Figure 6
Two curvature templates are provided with the apparatus. One contains arcs of circles with radii of curvature from 4 cm to 125 cm and the other contains radii from 100 cm to 650 cm . Radii of curvature of the tracks in the chamber are obtained by determining which of the arcs on the templates best match the curves made by the tracks.

The most straightforward, but not the best method to make this determination is to place the templates on the projection table and try to fit the best curve onto the projected image of the track. This method may be adequate for very curved tracks, but is not suitable for straighter ones. It will be found that many curves seem to fit the same track and that a determination of the best fit is difficult.

The following method has been found to yield far better results:

1. Obtain a very sharp, hard ( 3 H or 4 H ) pencil.
2. Place a blank piece of paper on the projection table so that the image of the tracks to be measured is projected onto the paper.
3. Making sure that the paper is flat and smooth, very carefully make three (recall that three points in a plane completely determine a circle) tiny, light dots, spaced out along the length of the track, using as much of the track as is plainly visible.
4. Be sure that the dots are placed, as nearly as possible, at the centre (width-wise) of the track. Putting the dots at either edge, rather than in the centre, especially for straighter tracks, can introduce an error as large as 100 cm in the determination of the radius (see Figure 7).

Figure 7

5. Take the paper off the projection table and, placing the template on top of it, try to find the curve along which all three of the dots lie. Then read off the corresponding radius (in cm ) from the edge of the template. This number gives the measured radius of curvature. Following sections will describe the corrections that must be made to this value to get the true radius of curvature.

The method outlined above will yield good results if followed carefully. A little practice will be rewarded in perfection of the technique. Choose one beam track which goes through the chamber without interacting and make four or five independent measurements of its radius of curvature using the technique outlined above. You should find that even the size of the dots will affect the measurement - the smaller the dots the more consistent the results. If you are able to make three or four consecutive determinations which agree with one another within a reasonable experimental uncertainty you may consider yourself proficient enough to proceed.

Hints, Warnings, and Reminders:

1. Keep your pencil very sharp - use a hard lead pencil.
2. Make your dots tiny and light.
3. Keep your dots in the middle, width-wise, of the track.
4. Use as much of the track as you can.
5. The best curve should pass through the centres of all three dots.
6. Once you think you have found the best curve, test those on either side of it to make sure that they are not as good a fit as the one you have chosen.

## Angles

As has been discussed in Section II we need to measure not only the total momentum of each track, but also the components of momentum in the $x$ and $y$ directions, where the $x$-direction is chosen to lie along the incident pion direction just before the pion interacts with a proton. We want to find the angle which each track makes with the $x$-direction as the track leaves the interaction vertex. This can also be done by using the dot method.

To measure the angle, simply make one dot at the vertex and then make one dot near the vertex on each track. It is suggested, for the purpose of measuring both the momenta and angles of all tracks in any event, that dots be made at positions roughly as shown in Figure 8 (dot size greatly exaggerated).

Figure 8


Dots 3,5 , and 7 and 4,6 , and 8 are to be used to find the radii of curvature of the two outgoing tracks. After the radii have been determined draw straight lines through dots 1 and 2, 1 and 3, and 1 and 4 as shown in Figure 9.


Figure 9
Using a protractor one can measure angles $\theta_{\mathrm{C}}$ and $\theta_{\mathrm{D}}$, the angles that the two outgoing tracks make with the x -axis.

NOTE: This method will yield good results only if points 2,3 , and 4 are close to the vertex. Notice that if a straight line is drawn between the vertex and a point far from it (point 8, for example), this straight line will not give the proper angle for the track. What is desired is the angle at which a particle leaves the vertex, and since the tracks curve in the magnetic field, points must be taken close to the vertex.

## Finding the Momentum Calculation Factor

In order to determine the momentum of any particle we need to know the radius of the almost (why not perfectly?) circular path it makes while traversing the bubble chamber. The radius of curvature we measure on the table, however, is not necessarily the radius of curvature in true space, since the image is not necessarily true size. Also, in spite of the fact that all tracks lie in planes roughly parallel to the $x-y$ plane, the curvature of a track as viewed from a camera will be somewhat distorted. This effect is due to the fact that a light ray from a given point in the chamber must pass through several media (hydrogen, glass, air), with different indices of refraction, before entering the camera. Although this distortion will vary somewhat at different positions in the chamber, it is sufficient for this experiment to assume that it is constant for all tracks.

The momentum calculation factor accounts for the magnification and optical distortion and enables the momentum to be determined from the measured radius of curvature. Each slide will have a different momentum calculation factor.

This factor is determined as follows:
The incoming beam kinetic energy is known to be $790 \pm 24 \mathrm{MeV}$. This corresponds to a beam momentum of $919 \pm 24 \mathrm{MeV} / \mathrm{c}$.

The momentum calculation factor (different for each slide) is determined by measuring the incoming beam momentum and comparing the result to the known value.

For each slide on which there is an interaction that you will be analyzing:

1. Measure the radii of curvature (in cm ) of a number ( 5 or so) of incident beam tracks.
2. Calculate the average measured radius of curvature, $R_{\mathrm{AVE}}$.
3. Calculate the average measured incident beam momentum, $p_{\text {AVE }}$ from

$$
\begin{aligned}
p_{\mathrm{AVE}} & =0.3 \times B \times R_{\mathrm{AVE}} \mathrm{MeV} / \mathrm{c} \\
& =0.3 \times 12.2 \mathrm{kG} \times R_{\mathrm{AVE}} \mathrm{MeV} / \mathrm{c} \\
& =3.66 \times R_{\mathrm{AVE}} \mathrm{MeV} / \mathrm{c}
\end{aligned}
$$

You will likely find that you do not obtain $919 \mathrm{MeV} / \mathrm{c}$. This is due to the magnification and the optical distortion.
4. The factor by which the measured momentum must be multiplied to obtain the true momentum is

$$
919 / p_{\mathrm{AVE}}
$$

5. The momentum calculation factor, $F$, the factor by which the measured radius of curvature of a beam track must be multiplied to obtain the true momentum, is thus:

$$
F=3.66 \times 919 / p_{\mathrm{AVE}}
$$

and since

$$
\begin{aligned}
& p_{\mathrm{AVE}}=3.66 \times R_{\mathrm{AVE},} \\
& F=3.66 \times \frac{919}{3.66 \times R_{\mathrm{AVE}}}=\frac{919}{R_{\mathrm{AVE}}}
\end{aligned}
$$

(Note that this result could have been obtained immediately, without any of the intermediate steps, by realizing that momentum is directly proportional to radius of curvature.)

## Momentum Determination from Range Measurements

When an interaction takes place most of the outgoing particles emerge from the vertex at high speeds and leave the chamber, going out either through the ends, the sides, or one of the glasses. However, occasionally a particle will emerge from an interaction with low enough energy so that, slowing down as it goes through the liquid hydrogen, it eventually comes to rest while still inside the chamber. When a particle does come to rest in the chamber there is a more accurate way of
finding its momentum than by measuring its radius of curvature. Very accurate theoretical calculations have been made which predict the length of path (range) of a particle of a given mass and velocity travelling through various media. These calculations have been verified experimentally.

Figure 10 is a range-momentum graph for various particles in liquid hydrogen of density 0.06 $\mathrm{gm} / \mathrm{cm}^{3}$ (close to the density of hydrogen in the bubble chamber). If the type of particle and its range are known, its momentum can be obtained from the graph. Among the events on the slides you will find several positive tracks which stop in the chamber. If a positive particle stops in the chamber without decaying into other particles, then it must be a proton. The reason for this is that the proton is the only stable positive particle. Hence, if we see a positive track which ends inside the chamber without decaying we know it must be a proton and can, by measuring its length, find its momentum with the help of the graph.

Recall as mentioned previously that images on the projection table are not true size. Therefore, before the range-momentum graph can be used, the magnification of the projection unit must be determined.

You should notice that in each interaction picture there are marks which look something like crosses. These are called "fiducial marks". The fiducial marks are etched onto the front and back glasses of the bubble chamber. Those on the front glass point in one direction, those on the back glass in the other. The locations of all fiducial marks are given in Figure 11. Since the coordinates of all fiducial marks are known, they provide a fixed frame of reference, a coordinate system with respect to which any point in the chamber may be located.

To find the magnification:

1. Measure the distance on the projection table between any two fiducial marks on the same glass. Call this distance $D_{\text {MEAS }}$.
2. Calculate this distance in real space from the data given in Figure 11. Call this distance $D_{\text {TRUE }}$.
3. The magnification, $M$, is then given by the ratio

$$
M=D_{\mathrm{MEAS}} / D_{\mathrm{TRUE}}
$$

4. To obtain greater precision it is suggested that an average be taken of several determinations of M using different slides and different sets of fiducial marks. Is $M$ expected to be the same for the front and back glasses of the bubble chamber? Take this into account when averaging your $M$ values.

Thus, to obtain true lengths in the bubble chamber, measured lengths must be divided by the magnification, $M$.

## Figure 10



Figure 11A
Front Glass Fiducial Positions


Figure 11B
Back Glass Fiducial Positions


All coordinates are in cm.

## SECTION IV - OUTLINE OF PROCEDURE AND DATA ANALYSIS

Experimental Constants:

```
proton mass = 938.3 MeV/c}\mp@subsup{}{}{2
neutron mass = 939.6 MeV/\mp@subsup{c}{}{2}
charged pion mass = 139.6 MeV/\mp@subsup{c}{}{2}
neutral pion mass = 135.0 MeV/\mp@subsup{c}{}{2}
incident beam kinetic energy = 790 MeV \pm3%
incident beam momentum = 919 \pm24 MeV/c
magnetic field = 12.2 kG
```

Total initial energy $=1868 \pm 24 \mathrm{MeV}$

1. Determine an average value for the magnification, $M$. (All the slides should be examined for their fiducial marks. In addition to the easily-found back glass fiducial marks, be sure to measure some front glass fiducial distances - slides 3,4 , and 6 have two front glass fiducial marks. The average value calculated for $M$ can be applied to all the slides, but heed the note on p. 15 regarding the best way to calculate this average value.)
2. Choose a slide to be analyzed.
3. Record the slide number.
4. Measure the radii of curvature of 4 or 5 non-interacting, incident negative pion tracks. Calculate the momentum calculation factor, $F$.
5. Measure the radii of curvature and angles of the outgoing tracks. Angles are defined to be positive if the track is "above" the incident direction, negative if "below". Remember to record which particle is positively charged and which is negatively charged. If the positive particle stops in the chamber, record its track length and whether or not it decays.
6. As was discussed in Section II, the events lend themselves to three hypotheses:
i) $\quad \pi^{-}+p \rightarrow \pi^{-}+p$
ii) $\quad \pi^{-}+\mathrm{p} \rightarrow \pi^{-}+\mathrm{p}+\pi^{0}$
iii) $\quad \pi^{-}+\mathrm{p} \rightarrow \pi^{-}+\pi^{+}+\mathrm{n}$
(elastic)
(inelastic)
(inelastic)
7. Using the relevant equations from Section II, is momentum conserved in the $x$ and $y$ directions within experimental uncertainty? (To answer, you must make some estimate of your experimental error, i.e. error calculations are required.)
8. Is total energy conserved assuming the event to be elastic?
9. Assume that a neutral pion is produced. Using the accepted neutral pion mass and requiring that momentum is conserved, is energy conserved within experimental uncertainty?
10. Assume that the reaction products are: $\pi^{-} . \pi^{+}, \mathrm{n}$

Using the accepted n mass and requiring that momentum be conserved, is energy conserved within experimental error?
11. All three hypotheses have been tested. Which hypothesis is most consistent with the data? Classify the event as being of type i), ii), or iii).
12. Repeat the above procedure (2. to 11.) for four other events on four different slides. (There is one event per slide.)
13. For all inelastic events, calculate the experimental mass of the neutral particle by requiring that momentum and energy be conserved (equation 28). Compare with the accepted mass.

An Excel spreadsheet, bubble.xls, is available to assist with performing the required calculations. The spreadsheet is obtained from the lab manual web page:
http://physics.usask.ca/~bzulkosk/modphyslab/phys381.htm.
Note that you are still required to show one complete set of sample calculations (including error calculations). In other words, you must completely analyze one interaction "by hand". For the other interactions, tabulate the pertinent information from the computer-calculated results, and explain how you used this information to decide which type of interaction occurred.

BEFORE attempting to use the spreadsheet, the following calculations must be performed:

- From your data determine an average value for the magnification.
- If you recorded the lengths of any non-decaying positive tracks, use the rangemomentum graph and your magnification to determine the momenta of these particles.
- From your data calculate the momentum calculation factor, $F$, for each slide on which you measured an interaction.
Enter your data in column B of the Data sheet. The data is automatically transferred to the Results sheet, which contains the results of most of the required calculations.

