The multiple scattering of 4·5 MeV electrons

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The multiple scattering of 4·5 MeV electrons by foils of aluminium, copper, molybdenum, silver and platinum has been determined, using a photographic method to measure the variation of scattered electron intensity with angle. It was found that the results were in good agreement with the Molière theory into the region of plural scattering, this being the upper limit of angles covered by the observations.

1. Introduction

There have been a number of experimental investigations into the angular distribution resulting from multiple scattering of fast electrons by foils. In particular, the work of Kulchitsky & Latyshev (1941), who measured the scattering of 2·25 MeV electrons from foils of elements with atomic numbers between those of aluminium and lead, should be mentioned, as well as that of Hanson, Lanzl, Lyman & Scott (1951) using 15·7 MeV electrons and foils of beryllium and gold. It was decided to make this present investigation using the 4·5 MeV electrons from the microtron (Henderson, Heymann & Jennings 1953), and, as little equipment was available at the time, to make the apparatus as simple as possible. Accordingly, it was decided to measure the variation of scattered electron intensity with angle using a photographic film held normal to the direction of the incident beam, the blackening of the film being measured with a microphotometer to give the relative intensities after calibrating the film. The self-consistency of the results was surprisingly good, an overall relative accuracy of better than 1 % being obtained. By this method the multiple scattering of electrons in foils of aluminium, copper, molybdenum, silver and platinum (2) was observed and the results were compared with the theory due to Molière.

2. Experimental arrangement

(*a*) *Primary beam*

The beam extracted from the microtron ($\sim 0{\cdot}2\,\mu$A average) has only a small angular spread and an energy spread of less than ± 1 % about the mean, this latter figure being obtained from later measurements using a double focusing sector magnet. As the beam is so nearly parallel it can be reduced to a very narrow beam by using a series of small defining holes as shown in figure 1, without too great a loss in intensity.

The electrons were extracted from the machine by means of an iron tube arranged so that its axis intercepts the desired orbit tangentially, the electrons passing straight down this tube, the inside of which is a field-free region (Henderson *et al.* 1953). The beam was first collimated at the entrance to the extractor by an aluminium plug (A) with a 0·042 in. axial hole. The plug at A also acts as an energy selector, the

energy spread of the emergent beam being reduced to less than $\pm 0{\cdot}2\,\%$. At the exit port of the machine a thick aluminium plate (B) with a 0·030 in. hole further collimated the beam. Electrons scattered from the second hole were almost completely removed by passing the beam through a 0·062 in. hole in plate C, whose position was carefully adjusted by means of horizontal and vertical screws operating through tombac bellows so that the core of the beam passed through centrally without touching the sides. The beam was sufficiently intense for these adjustments to be made using a fluorescent screen and a telescope. The adjustment was finally checked by letting the beam fall directly on to a piece of photographic film held in the position used for recording the patterns of the scattered electrons. The spot size in this plane was 0·17 cm in diameter.

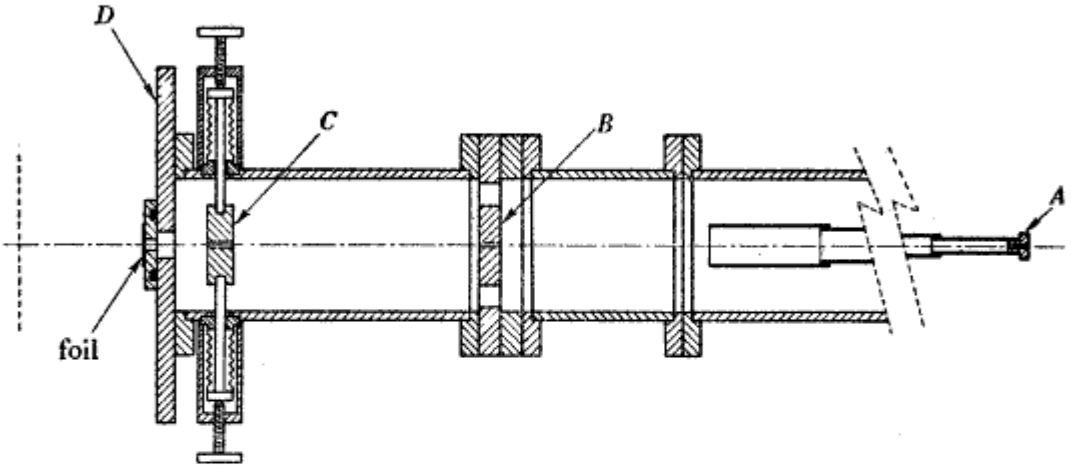

FIGURE 1. Beam extraction and collimation.

The energy of the electrons in the beam was deduced from the diameter of the final orbit in the machine and the value of the magnetic field, the latter having been calibrated by the nuclear resonance method. The value obtained for the kinetic energy was $4{\cdot}51 \pm 0{\cdot}04$ MeV. A photograph of the beam after it had been deflected by a strong permanent magnet brought near showed that the electrons had remained monochromatic and had not lost energy in passing through the collimating system, the recorded image remaining sharply defined.

(*b*) *Foils*

Foils of pure aluminium, copper, molybdenum, silver and platinum (2) with thicknesses chosen to give approximately the same angular distribution of scattered electrons were mounted on thick disks of brass so that they covered the small hole through the centre of each disk, being waxed at their edges to form a vacuum seal. Each disk had an O-ring groove on the side away from the foil, and was placed centrally on the plate D when it sealed off the collimating system and the machine from the atmosphere.

The foil position was adjusted by first dusting its outer surface with fluorescent powder and then sliding the brass disk on the plate D until the green spot which indicated where the electrons were striking the foil was central. The mount was then

tightly clamped to plate D, the fluorescent powder cleaned off the foil, and the distance of the foil from the front face of plate D measured with a depth gauge.

After the experiment the thickness of each foil was measured by cutting out the central portion of the foil (approximately 4 mm square) and determining its area by plotting out its shape using a Cooke microscope with micrometer movements on the mechanical stage and determining its weight on a balance measuring to $\pm 100\,\mu$g. The length measurements were standardized against a slip gauge and the weights against a 30 mg rider later calibrated against N.P.L. standardized weights. The thickness of each foil was determined in this way to better than $\pm 0{\cdot}5\,\%$.

(c) *Measurement of distribution of scattered electrons*

The scattered electrons were recorded in a camera consisting essentially of a large cylinder attached to plate D as shown diagrammatically in figure 2. The further end of the camera was closed using a 0·004 in. foil of aluminium in order to prevent back-scattered electrons. This foil was clamped between O-rings and was supported over its surface by a thin framework of brass. The film itself was clamped between two aluminium plates shaped as shown on the right of figure 2 so that the film was held in one plane; it was necessary to use film and not plates as the latter were found to give an appreciable amount of back-scattered electrons from the glass. The film

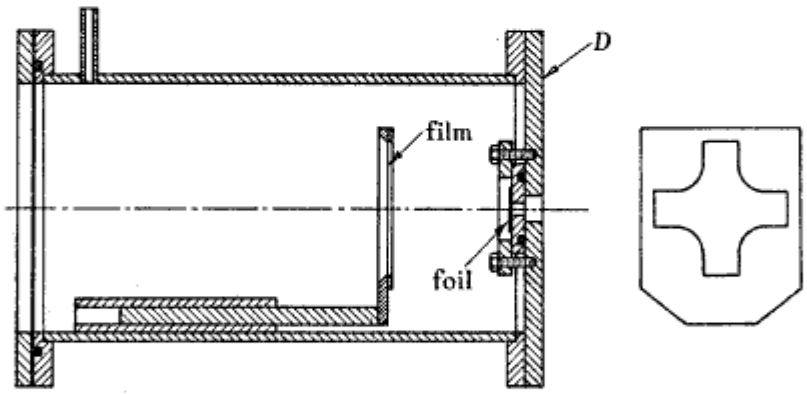

FIGURE 2. Camera.

holder was attached to a rod of square cross-section which slid into a mount on the base of the camera, its position being located by a click-stop mechanism. For each film loaded into the camera, the exact position of the aluminium carrier relative to the plane of the open end of the camera (i.e. essentially to plate D) was measured, again with the depth gauge, to determine the foil to film distance in each case. The location of the film in the carrier itself was marked each time by pricking through small holes in the front face of the carrier. The camera was not in communication with the main vacuum system and was pumped out for each exposure to a pressure of less than 0·05 mm using a single-stage rotary pump.

After processing, the films were microphotometered along two perpendicular lines through the centre of the pattern. In this way four measurements of the fall-off in photographic density from the centre were obtained. The agreement in each case was very close, and the average was used in the calculations.

(*d*) *Processing and calibration of the film*

The film finally chosen was Ilford line film because its response curve was found to be very nearly linear. All films were developed in a dish, with constant agitation, using fresh 1 D 19 developer for 5 min at 20° C, were then washed for 1 min and fixed in a standard hypo bath for 10 min.

As only a relative calibration of the film was required, a number of methods which do not require monitoring the electron beam could be considered.

One method which was tried made use of a large brass disk, near the periphery of which were marked a number of circles concentric with the axis of the disk. The first circle had one hole through the disk, the next two holes and so on up to ten. With no apparatus on the front of the machine, a fairly thick foil was placed over the exit port. The electrons from the machine were scattered by this foil, and also in the air, so that the electron density over any small area 3 ft. in front of the machine was fairly uniform. The disk was rotated at this point, and a piece of film was

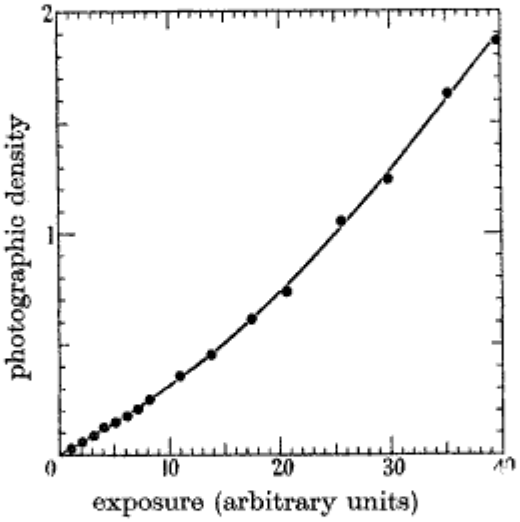

FIGURE 3. Line-film calibration curve.

placed immediately behind and overlapping the edge of the disk. In this way a set of ten arcs was recorded corresponding to relative exposures of 1 ... 10 units. Unfortunately, this method had to be discarded because it was found that the effective aperture varied from hole to hole, owing to slightly different scattering from their sides.

The method finally adopted required a monitor in the primary beam. An ionization chamber was placed over the exit port of the machine, and the ionization charge was recorded using an automatic Miller integrating circuit. The film was placed behind a thick aluminium bar through which a single $\frac{1}{4}$ in. hole had been drilled, the bar being 3 ft. in front of the exit port. A series of exposures was made, the film being moved between exposures, so that a series of disks was obtained on developing the film.

In all the microphotometer measurements, the densities were expressed relative to the reading for a particular piece of film that had not been exposed but had been

developed in the standard manner. By this means it was possible to allow for the small amount of overall blackening caused by X-rays during the calibration and to check that the X-ray blackening during the actual experiment was negligible.

The spurious overall blackening of the two calibration films was allowed for in the following manner. Taking first the film with a set of short exposures, the density of each disk was determined (values up to 0·25) as well as the density of the background (0·005), all readings being relative to the standard zero-density film. The additional exposure necessary to make this curve pass through the origin was found by extrapolating the curve and this set of results so corrected. The second film had densities up to 1·8 and an overall background density of 0·039 which could be allowed for by reading off the corresponding exposure from the curve for the first film. The final calibration curve for the line film is shown in figure 3.

3. Corrections to results

(*a*) *Correction for plane film*

The theory of multiple scattering used here expresses the intensity in terms of the number of electrons falling normally on to unit area of the inner surface of a sphere whose centre is at the scattering foil. Corrections are necessary to the distribution obtained in our case from measurements on a flat piece of photographic film arranged normal to the direction of the primary beam. Two corrections are necessary.

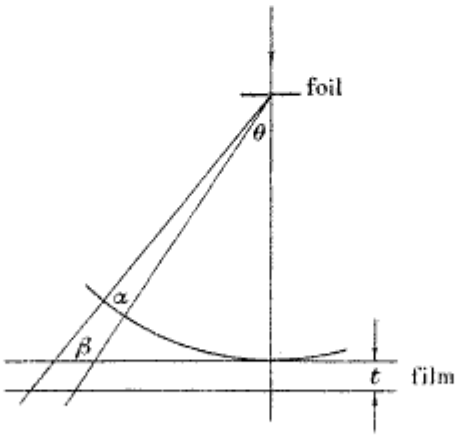

Figure 4

(i) Referring to figure 4, the number of electrons per unit area of the film (at β) is equal to the number of electrons per unit area of the sphere (at α) times $\cos^3\theta$.

(ii) The film is calibrated with electrons which enter normally, and traverse a thickness t of emulsion ($\sim 25\,\mu$). In the multiple-scattering measurements, the thickness traversed is $t/\cos\theta$, the number of blackened grains is increased by $1/\cos\theta$, so that the exposure deduced from the calibration curve must be multiplied by $\cos\theta$ to allow for this effect.

Taking both corrections into account the exposures as found from the microphotometer readings and calibration curves must be multiplied by $1/\cos^2\theta$.

(*b*) *Spot size*

The cross-section of the beam at the plane of the film when the scattering foil was absent was found to be a disk of diameter 0·17 cm. Assuming the intensity to be uniform across the section of the beam (diameter $2a$) the correction to be applied is given by

$$-100\left\{\frac{1}{x}\frac{\mathrm{d}I}{\mathrm{d}x}+\frac{\mathrm{d}^2I}{\mathrm{d}x^2}\right\}a^2/8I\,\%$$

where x is the distance measured along the film from the centre of the pattern and I is the corresponding intensity. This correction is less than 0·1 % at any point of the curves, and so could be disregarded. Similarly, the spot size of the light beam in the microphotometer at the film did not introduce any appreciable errors.

(*c*) *Energy loss in the foil*

The calculated energy loss in the foil varied from 2·2 % for aluminium to 1·0 % for the thin platinum foil. It can be shown that for small energy losses and small angles the theoretical curve may be corrected by increasing the values of the multiple scattering angle θ by an amount $\theta\,\mathrm{d}E/E$, where E is the initial energy of the electrons and $\mathrm{d}E$ is half the energy loss in the foil. This correction, of the order of 1 %, was just significant and the curves were corrected.

(*d*) *Small-angle corrections*

The Molière theory is accurate for small angles only, but it can be shown that, in our case, in determining the 1/e point, a suitable correction (Bethe 1953) is to multiply the calculated intensity by $(\theta/\sin\theta)^{\frac{1}{2}}$. This changes the theoretical angle for the 1/e value of the distribution by less than 0·2 % which is within experimental error and was neglected.

4. Results and comparison with theory

The results for the six foils are shown in figure 5. The lines represent the multiple scattering to be expected according to the theory of Molière as given by Bethe (1953), and the points are the experimental values obtained, these points being normalized at $\theta = 0$.

The theory gives an expression for $f(\theta)$, the distribution function for the number of electrons scattered into the angular range between θ and $\theta + \mathrm{d}\theta$ with the direction of the primary beam of the form

$$f(\theta)\,\mathrm{d}\theta = \vartheta\,\mathrm{d}\vartheta[f^{(0)}(\vartheta)+B^{-1}f^{(1)}(\vartheta)+B^{-2}f^{(2)}(\vartheta)+\ldots]. \qquad (1)$$

$\vartheta = \kappa\theta$, where κ depends upon the energy of the electrons and the details of each particular foil, while B depends on the number of collisions each electron has, on the average, in traversing the foil. The functions $f^{(0)}$, $f^{(1)}$, etc., are defined in the paper, and for different values of ϑ values of these functions are tabulated. The $f^0(\vartheta)$ term is the Gaussian function of the earlier theories. $B^{-1}f^{(1)}(\vartheta)$ corresponds to single scattering, and the higher-order terms are correcting terms. As ϑ increases,

the Gaussian term decreases exponentially until finally the single scattering term is predominant, the higher-order terms still remaining relatively unimportant. Thus the multiple scattering for all angles can be represented quite accurately by the first three terms, $f^{(0)}(\vartheta)+B^{-1}f^{(1)}(\vartheta)+B^{-2}f^{(2)}(\vartheta)$.

In this experiment, the second and third terms were of the order 5 and 2% of the Gaussian at $\theta = 0$, and at the largest angles the first two terms were approximately equal, the third still being approximately 2% of them. The agreement

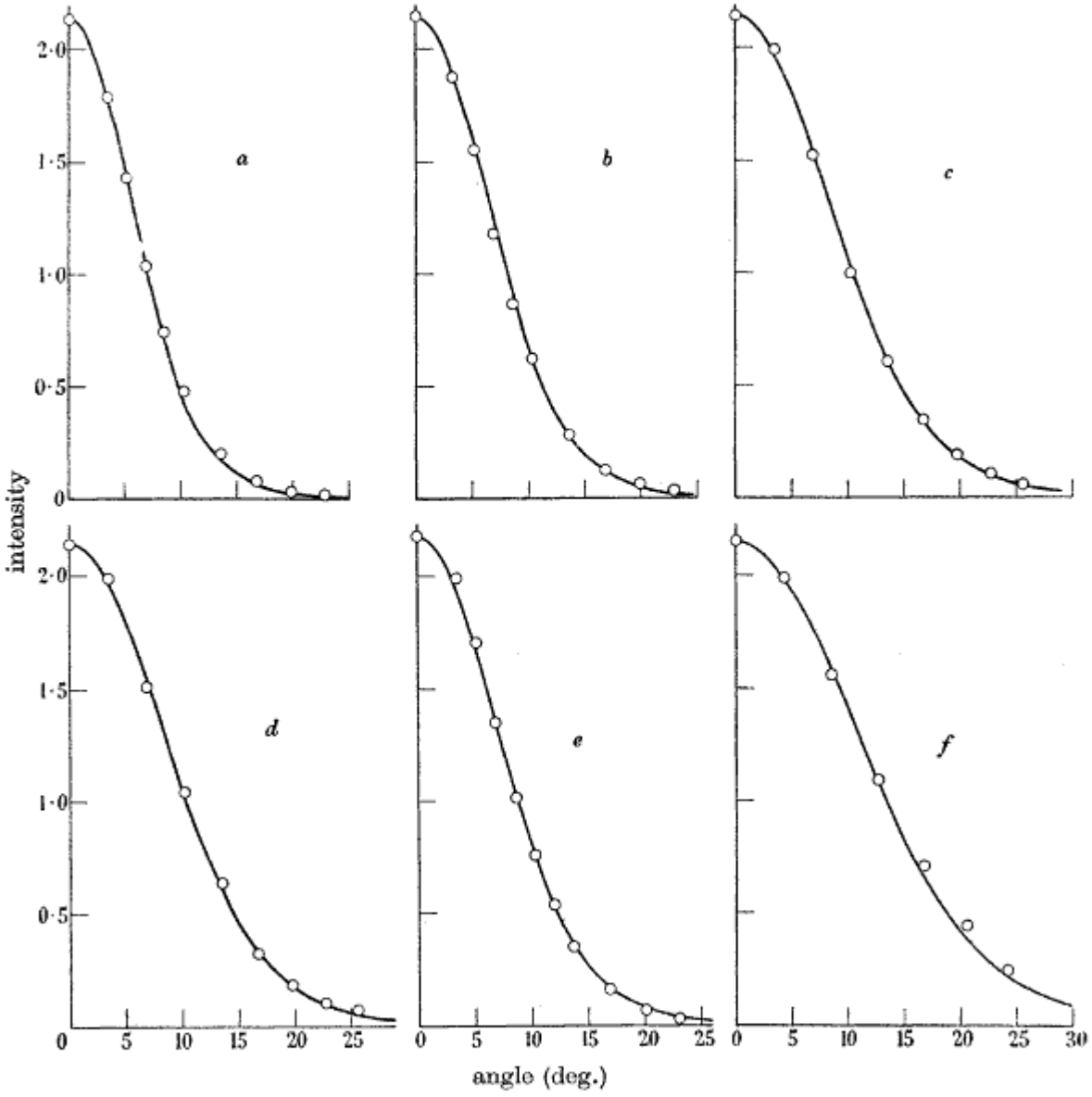

FIGURE 5. (*a*) Aluminium; (*b*) copper; (*c*) molybdenum; (*d*) silver; (*e*) platinum (thinner foil); (*f*) platinum (thicker foil). Lines, Molière theory; ○, experimental values.

between the experimental results and the theoretical curves is good up to the plural scattering region, where the results are slightly high. The accuracy at the larger angles is not sufficient, however, for us to attach any particular significance to this deviation. With the apparatus and foils used it was not possible to extend the readings into the single scattering region.

In table 1 the experimental values for the angle in degrees at which the intensity has fallen to 1/e of that at $\theta = 0$ are compared with the values given by theory. ω_m is the width given by equation (1), ω_g that obtained if only the first or Gaussian term of this expression is used, and ω_H that obtained using a slightly narrower Gaussian curve as suggested empirically by Hanson *et al.* to represent the distribution of scattered electrons without any additional terms.

It is seen that the agreement between theory and experiment is good, the difference between $\omega_{\text{obs.}}$ and ω_m being less than 1 % except for copper (1·5 %) and the thicker platinum foil (2·9 %).

TABLE 1

foil	thickness (mg/cm²)	Molière theory constants B	χ_c	ω_m	ω_g	reduced Gaussian ω_H	observed width $\omega_{\text{obs.}}$	fractional error $\frac{\omega_{\text{obs.}}-\omega_m}{\omega_m}$
aluminium	65·2	8·44	0·0532	8·16°	8·86°	8·20°	8·23°	+0·009
copper	44·4	8·04	0·0622	9·28°	10·10°	9·32°	9·14°	−0·015
molybdenum	53·2	8·14	0·0801	12·02°	13·09°	12·08°	11·98°	−0·003
silver	48·4	7·98	0·0804	11·92°	13·00°	11·99°	11·96°	+0·003
platinum	26·8	6·94	0·0733	9·97°	11·06°	10·06°	9·89°	−0·008
platinum	54·4	7·76	0·1049	15·30°	16·73°	15·37°	15·76°	+0·029

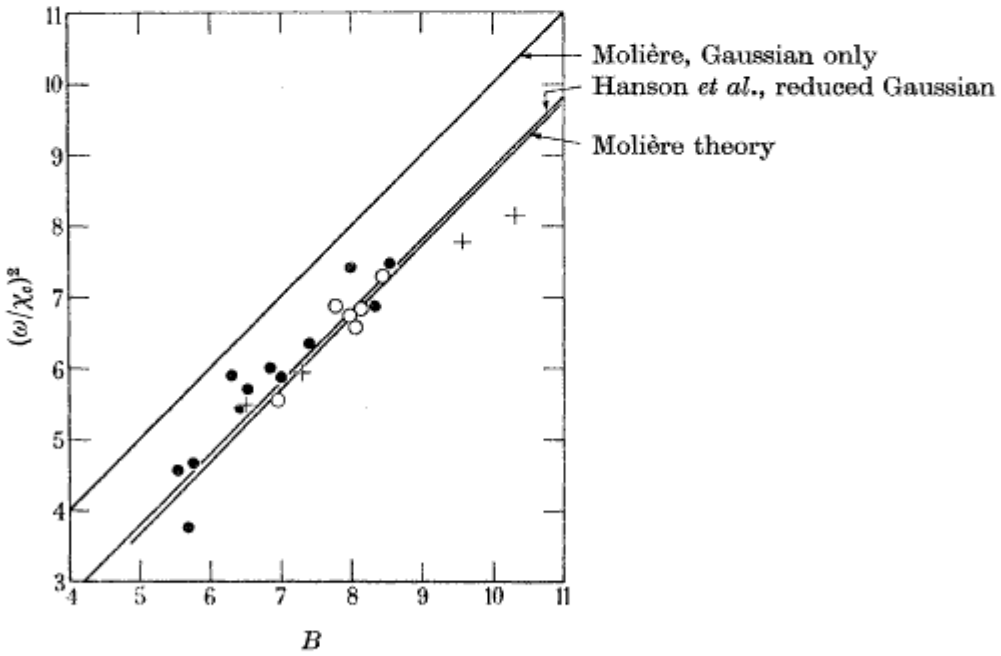

FIGURE 6. $(\omega/\chi_c)^2$ plotted against Molière's constant, B. ●, Kulchitsky & Latyshev (2·25 MeV); +, Hanson *et al.* (15·7 MeV); ○, present results (4·5 MeV). The results include a range of elements from beryllium to lead.

Using the narrower Gaussian curve suggested by Hanson *et al.*, the angle ω where the intensity has dropped to 1/e of the central value can be expressed by the equation $\omega^2 = \chi_c^2(B - 1{\cdot}2)$, where χ_c is Molière's unit probability angle and B depends on the number of collisions. $(\omega/\chi_c)^2$ has been plotted against B in figure 6,

and included on this curve are the results of Kulchitsky & Latyshev at 2·25 MeV and Hanson *et al.* at 15·7 MeV. It is seen that the agreement for all three sets of results is good.

References

Bethe, H. A. 1953 *Phys. Rev.* **89**, 1255.
Hanson, A. O., Lanzl, L. H., Lyman, E. M. & Scott, M. B. 1951 *Phys. Rev.* **84**, 634.
Henderson, C., Heymann, F. F. & Jennings, R. E. 1953 *Proc. Phys. Soc.* B, **66**, 654.
Kulchitsky, L. A. & Latyshev, G. D. 1941 *Phys. Rev.* **61**, 254.