

Track Structure

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Introduction

Track structure relates to the manner in which energy is deposited by tracks of charged particles as they slow down. As we have seen, the rate of energy lost by a charged particle per unit length (LET_{∞}) along its track depends on such physical quantities as its mass, velocity and charge. It is not surprising that at the *same* LET_{∞} , different particles (e.g., protons, He and Ne ions) have different patterns of energy deposition. In general, at the same LET_{∞} , heavier particles spread out their energy loss to greater distances around the track trajectory than lighter particles. Since a majority of the radiation within space travelers will consist of high-energy charged particles (primaries and secondaries) from the galactic and solar particle radiation, it is important to determine the difference, if any, in the biological effects caused by various particles at the same LET_{∞} . Since radiation risk is calculated by using a function, the Quality Factor, which depends *only* on the particle's LET_{∞} , such calculated risks may be in error if different particles with the same LET_{∞} cause significantly different *biological* effects. We now examine several ways of looking at the track structure of the charged particles that are important in the radiation environments to be encountered by space travelers.

A Simple Model of Track Structure

We recall that the LET_{∞} or dE/dx of a charged particle varies approximately as z^2/β^2 where z is the particle's charge and β is the ratio of the particle's velocity to that of light. In a simplified theory of track structure, the energy deposition pattern around the track trajectory can be thought of as consisting of a "core" and a "penumbra". The energy deposited in the "core" (i.e., close to the trajectory) is from excitations and collective oscillations (called "glancing" collisions) of atoms very close to the track, as well as from ionizations ("close collisions") causing *delta rays* (electrons knocked out of the atoms), that travel away from the trajectory. The latter are events in which energy is transferred directly to the electrons of the atoms of the medium being traversed. The maximum energy transferable to a delta ray depends on the energy and mass of the charged particle and can be calculated by applying principles of conservation of energy and momentum. The delta rays can have considerable energy and tend to travel away (i.e., scatter) from the trajectory, thus depositing energy at distances some distance from the particle trajectory. This part of the energy loss pattern is called the "penumbra".

A simple model to describe this penumbra was introduced into radiobiology by Butts and Katz (1967) in studying the inactivation of dry enzymes and viruses by heavy ions. To obtain the way the radiation (actually the absorbed energy density or "dose") decreases with distance from the track trajectory, they started with the well-known Rutherford delta-ray energy distribution formula (see Rossi (1952), for example) giving the number of delta rays per unit length of track having energies between w and $w + dw$ produced by a charged particle of effective charge $z*e$ moving with speed βc :

$$dn = (Cz^2/\beta^2) dw/w^2 \quad w \leq w_{max} \quad (1)$$

$$dn = 0 \quad w > w_{max}$$

where $C = 2\pi Ne^4/mc^2$ and N is the number density of electrons in the stopping medium, e and m are the electron charge and mass. The maximum energy transferable to an electron, w_{max} is equal to $2mc^2\beta^2\gamma^2$, with $\gamma^2 = (1 - \beta^2)^{-1}$, and can be derived from kinematic considerations applying to a head-on collision between the particle and an electron. The “effective” charge on the particle, z^*e , is the atomic number of the particle times the electronic (or protonic) charge, ze , except at very low energy when there is electron charge pickup (capture) by the particle thus decreasing its effective charge just before it stops. It can be shown that electrons of energy w are ejected at an angle θ to the trajectory by the expression:

$$\cos^2\theta = w/w_{max} \quad (2)$$

From equation (1), we see that most of the electrons have energies much less than w_{max} and so have ejection angles approximately equal to 90° to the track direction. Thus, in this model, the electrons are assumed to be ejected normal to the track trajectory.

Now consider the energy deposition pattern looking head-on into the particle track. We proceed by calculating the energy per unit volume deposited by the delta rays (electrons) knocked-out of atoms of the medium by a charged particle of charge Z^*e and mass m traveling at velocity βc . The number of electrons (per unit track length) traversing a cylindrical shell of thickness dr and radius r centered on the track trajectory is the number with energy between $w(r)$ and w_{max} :

$$n[w(r)] = \frac{Cz^{*2}}{\beta^2} \int_{w(r)}^{w_{max}} dw/w^2 \quad (3)$$

$$= \frac{Cz^{*2}}{\beta^2} \left[\frac{1}{w(r)} - \frac{1}{w_{max}} \right]$$

The assumption is made that the electrons move in straight lines normal to the track trajectory and that the range-energy relation for the electrons is linear with energy. The latter is true within 10% at energies below 2 keV. Then $r = kw$ and $dw = k^{-1}dr$. For a cylindrical shell of thickness dr , the total energy deposited in the shell (per unit track length) is the number of electrons traversing the shell (per unit track length) times the energy deposited in the shell per electron. If the particle track length considered, ΔT , is long enough for many electrons to be ejected and short enough that the velocity of the particle can be considered constant, the total energy absorbed per unit mass (or “dose”) at a distance r from the trajectory can be obtained by calculating the total delta electron energy deposited at r divided by the mass of the cylinder (assumed here to be water):

$$D_\delta(r) = \frac{\Delta E}{\Delta m} = \frac{n[w(r)]dw\Delta T}{2\pi\rho_{water}rdr\Delta T} = \frac{Cz^{*2}k^{-1}dr}{2\pi\rho_{water}rdr\beta^2} \left[\frac{1}{w(r)} - \frac{1}{w_{max}} \right]$$

$$= \frac{Cz^{*2}}{2\pi\rho\beta^2} \left[\frac{1}{r} - \frac{1}{R} \right] \quad (4)$$

where we have set $\rho_{water}=1 \text{ gm/cm}^3$ and $R = kW_{max}$. For $r \ll R$, the second term can be neglected and the “dose” falls off as the reciprocal of the square of the distance from the trajectory. Note that particles with the same z^2/β^2 in this model have the same deposition patterns caused by these delta rays. So to the extent that this model adequately describes track structure (i.e., the energy loss patterns) of high energy charged particles, and if biological effects depend mainly on this “dose” caused by delta rays (electrons) being knocked out of atoms as the particles slow down, biological effects might be expected to be the same for all high energy charged particles with the same z^2/β^2 .

This energy from delta rays, however, is only part of the energy lost by charged particles. There is an “equipartition” rule that states that half the energy loss comes from the “glancing” collisions of the particle with the atoms of the medium. These collisions transfer small amounts of energy mostly by excitation of individual atoms and in collective oscillations of electrons. These two mechanisms occur predominantly very close to the track trajectory. This region where these mechanisms dominate has been called the track “core”. Early theoretical studies determined the physical extension of the core region (Chatterjee and Schaefer, 1976). In water, the core radius in micrometers was given as:

$$r_c = 0.00116\beta \quad (5)$$

The corresponding radius of the penumbra in micrometers is given as:

$$r_p = 0.768E + 1.925\sqrt{E} + 1.257 \quad (6)$$

where E is the kinetic energy of the particle in MeV/nucleon. This is an empirical expression that holds for energies greater than 2 MeV/nucleon. It takes into account the scattering of the electrons as they exit the core region.

In order to visualize the differences in energy deposition patterns for different particles with the same LET_∞ , these authors calculated the percentage of LET_∞ deposited within a radial distance as a function of that distance from the track trajectory for neon ($z = 10$), argon ($z = 18$) and iron ($z = 26$) ions. This is shown in Figure 1. The LET_∞ is $800 \text{ keV}/\mu\text{m}$ for all three particles. We note that the heaviest ion's (iron) energy deposition spreads out considerably farther than the lighter ions'. The neon ion's pattern is totally contained within $1 \mu\text{m}$, while for the iron ion, 10 % of the energy deposition reaches beyond $4 \mu\text{m}$. The physical parameters for the three particles of this LET_∞ are shown for comparison in Table I. Note that the velocities of the Ne and Fe ions vary by about a factor of three and the penumbra radii vary by a factor of forty-five. The core LET 's are about the same for the three ions.

same LET_{∞} Table I

Element	Ne	Ar	Fe
Z number	10	18	26
Speed, $\beta = v/c$	0.096	0.20	0.32
Energy, MeV/nucleon	4.4	20.0	51.0
Core radius r_c , microns	0.0011	0.0023	0.0037
Penumbra radius r_p , microns	0.6	7.7	27.0
LET_{∞} , keV/micron T	800	800	800
Core LET, KeV/micron T	430	423	420

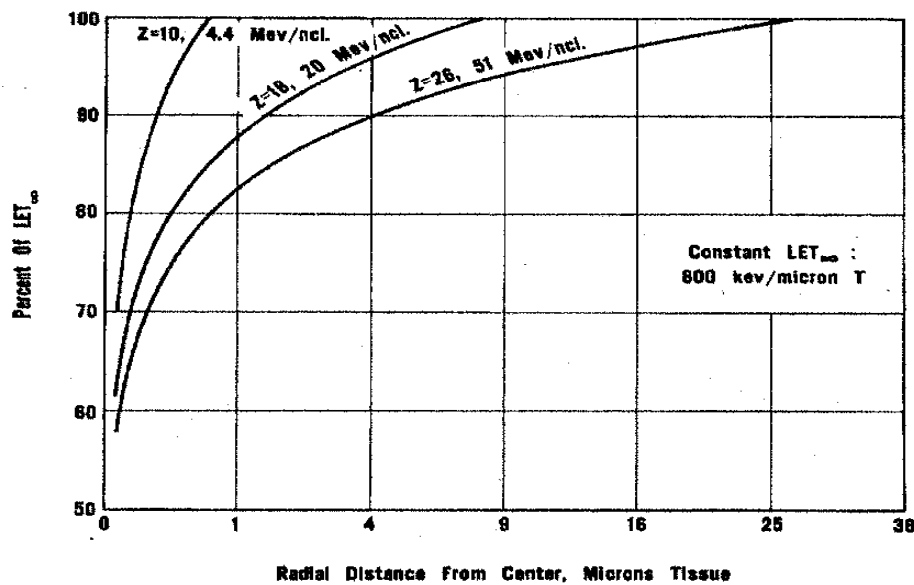


Fig. 1 Percentage of LET_{∞} deposited within a radial distance from the track trajectory plotted as a function of that distance for neon ($z=10$), argon ($z=18$) and iron ($z=26$) ions, all with total $LET_{\infty} = 800 \text{ keV}/\mu\text{m}$ (from Chatterjee and Schaefer, 1976). Note the non-linearity of the x-axis.

References

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- Chatterjee, A. and H. J. Schaefer, Microdosimetric Structure of Heavy Ion Tracks in Tissue, *Rad. and Environm. Biophys.* **13**, 215-227 (1976).
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