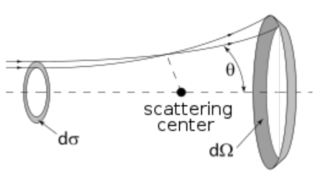
PHYS 352

Charged Particle Interactions with Matter

Intro: Cross Section

- **cross section σ –** describes the probability for an interaction as an area
- **flux F –** number of particles per unit area per unit time



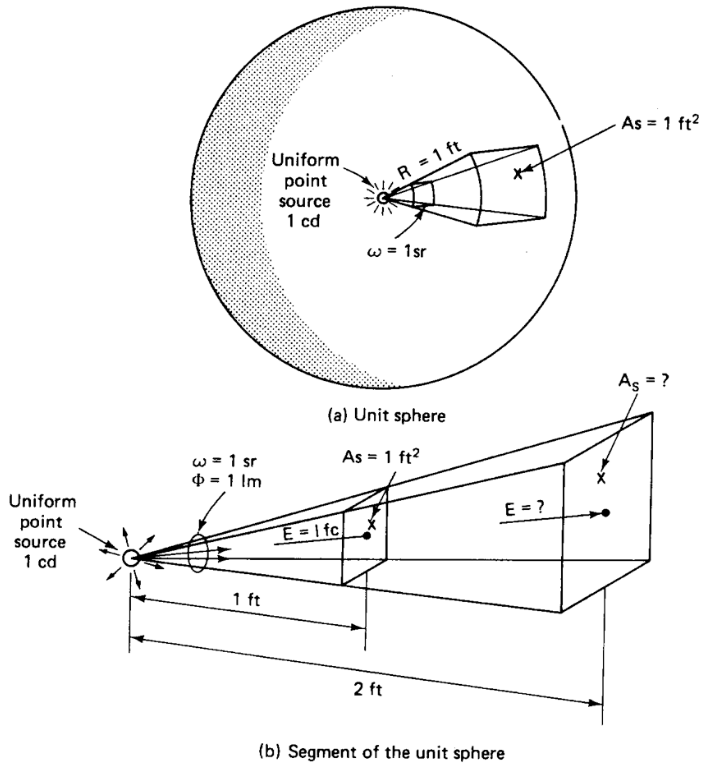
$$\frac{d\sigma}{d\Omega}(\theta,\phi) = \frac{1}{F}\frac{dN_s}{d\Omega}$$

$$dN_s = F\, d\Omega\left(\frac{d\sigma}{d\Omega}(\theta,\phi)\right)$$

- read the equation like this: "the number of particles that scatter, N_s, into a portion of solid angle per unit time is equal to the flux of incident particles per unit area per unit time multiplied by the probability (represented by a cross section area) that would scatter into that portion of solid angle"
- integrate over all solid angle to get the total cross section $\sigma = \int d\Omega \frac{d\sigma}{d\Omega}$
  - that's the representative cross section area for the interaction to cause any scatter (note: concept also applies to an absorption cross section and it doesn't have to actually scatter)

Solid Angle

- in 3D, a measure of the angular size accounting for both $\Delta\Theta$ and $\Delta\phi$
- "angular area" = surface area subtended divided by radius
- total solid angle around a point in space is 4π steradians
  - total angle around a point on a plane is 2π radians
- units are steradian [sr] (but it is actually unitless)



Intro: Surface Density

- **target number density n_{target}** – # of targets per unit volume
- thus $\mathbf{n_{target}\ \sigma\ \Delta x}$ is the probability that a single particle will interact while passing through that volume, where Δx is the thickness of the material
- for interaction of radiation with matter, useful to use mass thickness units rather than normal length units
  - mass thickness = $\rho\ \Delta x$ [g/cm^2]
- thus, if radiation passes through 2 g/cm^2 of air at 10 atm (ρ = 0.012 g/cm^3) or 2 g/cm^2 of water, the effect is roughly the same even though it went through a thickness of 1.67 m of air and a path length of just 2 cm through water
  - [g/cm^2] multiply by σ [cm^2], divided by [g/mol] and multiply by Avogadro's to get the interaction probability ($n_{target}\ \sigma\ \Delta x$)
- order of magnitude estimate: $\sigma \sim 10^{-24}$ cm^2 if 1 g/cm^2 thickness has an interaction probability of order unity (for 1 g/mol)

special unit for cross section: 1 barn = 10^{-24} cm^2

Passage of Charged Particles through Matter

- two things happen: a) the particle loses energy traversing matter and b) particle is deflected from its initial direction
- two main processes cause this: 1) <u>inelastic collisions with atomic electrons</u> in the material and 2) <u>elastic scattering off nuclei</u>
- other processes also cause energy loss: 3) bremsstrahlung, 4) emission of Cherenkov radiation (relative of bremsstrahlung), 5) nuclear reactions (rare, lower probability)
- makes sense to separate the consideration of heavy charged particles and light charged particles (i.e. electrons)
    - heavy particles don't undergo 3) and 4); 5) is rare; 2) is again less common compared to 1)...and heavy particles don't deflect much off electrons
    - basically need only consider inelastic collision with atomic electrons for energy loss of heavy charged particles

Heavy Charged Particles (like alphas)

- consider the maximum energy lost in each particle-electron collision

$M \bigcirc\!\!\longrightarrow v \quad \bullet\, m_e \qquad\qquad \bullet\, m_e \longrightarrow 2v$

- maximum $E_{loss} = 1/2\, m_e\, (2v)^2 = 1/2\, m_e\, 4\, (2E/M) = 4\, m_e/M\, E$
- if M is the proton mass that's 1/500 E
- if M is the mass of an alpha that's 1/2000 E
- hence, each collision causes only a small energy loss; number of energy loss events per macroscopic path length is large and hence the statistical fluctuations are small
    - can consider an average energy loss per unit path length dE/dx
- energy loss by nuclear collisions is less important because nuclei are much heavier than electrons (cause scatter but not energy loss) and because electron collisions are much more frequent

there are exceptions, like alpha particles scattering off hydrogen nuclei a charged particle zips by an electron and Coulomb force deflects the electron

Classical Energy Loss by Heavy Charged Particles

- note: maximum momentum transfer $\Delta p = m_e(2v) = 2\,(m_e/M)\,P$, is small so justifies the undeflected path assumption
- by symmetry, x-component impulse = 0
- electron receives y-component momentum impulse from Coulomb force

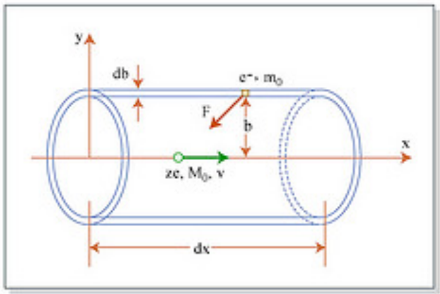
$$I = \int F\,dt = e\int E_y\,dt = e\int E_y \frac{dt}{dx}dx = e\int E_y \frac{dx}{v}$$

- Gauss' Law for an (infinitely long) cylinder gives

$$\int E_y 2\pi b\,dx = \frac{ze}{\varepsilon_0}; \quad \int E_y dx = \frac{2ze}{4\pi\varepsilon_0 b} = k\frac{2ze}{b}; \quad I = k\frac{2ze^2}{bv}$$

- energy gained by electron at distance b is

$$\Delta E(b) = \frac{I^2}{2m_e} = \frac{k^2 2z^2 e^4}{m_e v^2 b^2}$$



Energy Loss by Heavy Charged Particles cont'd

- let N_e be the density of electrons; energy lost to all electrons in slice between b and b+db in thickness dx is

$$-dE(b) = \Delta E(b) N_e dV = \frac{k^2 2z^2 e^4}{m_e v^2 b^2} N_e 2\pi b\,db\,dx = \frac{k^2 4\pi z^2 e^4}{m_e v^2} N_e \frac{db}{b} dx$$

- integrate over "all" physically meaningful values of b

$$-\frac{dE}{dx} = \frac{k^2 4\pi z^2 e^4}{m_e v^2} N_e \ln\frac{b_{max}}{b_{min}}$$

- classical maximum energy transfer is $\frac{1}{2}m_e(2v)^2 = \frac{k^2 2z^2 e^4}{m_e v^2 b_{min}^2}; \quad b_{min} = \frac{kze^2}{m_e v^2}$
- perturbation on the atomic electron caused by the passing heavy charged particle must take place in a short time compared to period 1/ω, the average orbital frequency of the bound atomic electron
- interaction time is t ≃ b/v $\quad b_{max} = \dfrac{v}{\omega}$

Energy Loss by Heavy Charged Particles cont'd

- the classical energy loss formula (derived by Bohr) is:

$$-\frac{dE}{dx}=\frac{k^2 4\pi z^2 e^4}{m_e v^2} N_e \ln\frac{m_e v^3}{\omega k z e^2}$$

- assumptions here are: classical, non-relativistic, particle must be heavy compared to m_e, interaction time is short thus the electrons are "stationary", does not account for binding of atomic electrons
- Bethe solved this using QM Born approximation, includes relativistic, still valid only for incoming velocity larger than orbital electron velocity, considers binding energy
- ...and the result is basically the same expression as above (I've collected terms together)

$$-\frac{dE}{dx}=\frac{4\pi k^2 e^4}{m_e c^2}\frac{z^2}{\beta^2}\frac{\rho Z N_A}{A}B(v); \quad B(v)=\ln(\frac{2m_e v^2}{I})-\ln(1-\beta^2)-\beta^2$$

Bethe-Bloch Energy Loss Formula

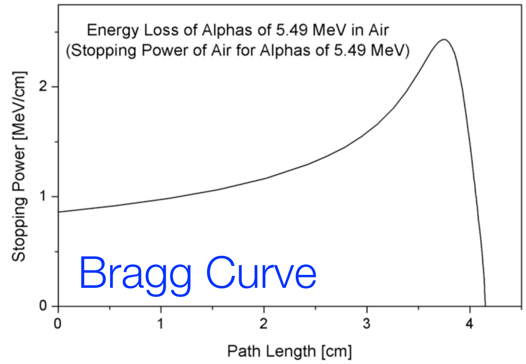
- actually B(v) takes on several forms depending on how many corrections one adds (e.g. by Bloch, Fermi added a density effect correction); B(v) contains the atomic physics and relativistic effects

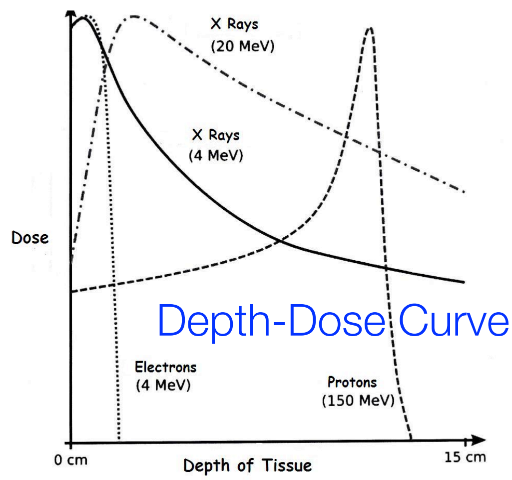
$$-\frac{dE}{dx}=\frac{4\pi k^2 e^4}{m_e c^2}\frac{z^2}{\beta^2}\frac{\rho Z N_A}{A}B(v)=4\pi r_e^2 m_e c^2\frac{z^2}{\beta^2}\frac{\rho Z N_A}{A}B(v)$$

- r_e is the classical electron radius $ke^2/(m_e c^2) = 2.818 \times 10^{-13}$ cm
- and for 1 g/mol $4\pi N_A r_e^2 m_e c^2 = 0.307\,\mathrm{MeV\,cm^2/g}$
- we define the stopping power: $\longrightarrow$ $-\frac{1}{\rho}\frac{dE}{dx}=[4\pi r_e^2 m_e c^2 N_A]\frac{z^2}{\beta^2}\frac{Z}{A}B(v)$
  - remember mass thickness = ρ Δx [g/cm^2]
- most materials have the same Z/A
- thus, apart from B(v) corrections, the energy loss for heavy charged particles goes as $\boxed{\frac{z^2}{\beta^2}}$

z^2 and $1/\beta^2$

- alpha particles have 4 times energy loss
- as the particle slows, the energy loss per unit length increases; right at the end, the ionization density is high
  - interesting for proton radiation therapy
- concept of "minimum ionizing particle"
  - especially cosmic ray muons, $m_\mu = 207\ m_e$, heavy but relativistic
- particle mass does not appear explicitly (but affects the velocity β)
- "alphas" has high ionization because they are heavy is actually because they are slow (slow because they are heavy)





Back to B(v)

$$-\frac{dE}{dx} = \frac{4\pi k^2 e^4}{m_e c^2}\frac{z^2}{\beta^2}\frac{\rho Z N_A}{A}B(v); \quad B(v) = \ln(\frac{2m_e v^2}{I}) - \ln(1-\beta^2) - \beta^2$$

- a full expression has $B(v) = \frac{1}{2}\ln(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}) - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C(I,\beta\gamma)}{Z}$
  - I is the mean ionization/excitation potential
    - this is like the orbital frequency ω in the Bohr formula
    - sets the criterion for short interaction; if particle is too slow (not energetic enough) its perturbation on the orbital electrons is adiabatic
  - W_{max} is the relativistic maximum energy transfer produced by head-on collision, properly accounting for finite M and m_e

$$W_{\max} = \frac{2m_e v^2\gamma^2}{1 + 2\frac{m_e}{M}\sqrt{1+\gamma^2\beta^2} + (\frac{m_e}{M})^2}; \quad \text{if } M \gg m_e, W_{\max} = 2m_e v^2\gamma^2$$

  - δ is the density effect – electric field of the particle tends to polarize the atoms in the material; far electrons "screened" from the particle
    - important at high energy (larger b_{max}); depends on density of material

Ionization Potential, Shell and Density Correction

$$B(v) = \frac{1}{2}\ln\left(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}\right) - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C(I,\beta\gamma)}{Z}$$

- C is the shell correction – when the velocity is small, the assumption that orbital electrons are stationary breaks down; correction is small in the range of interest
- there are parameterizations for I, δ, C based upon: averaging oscillator strengths of atomic levels to determine average ionization potential, using the plasma frequency of the material (for the density effect), etc.
- ultimately, can just measure the energy loss in different materials and parameterize the corrections (semi-empirical formulae) or just tabulate the values for I, δ, C

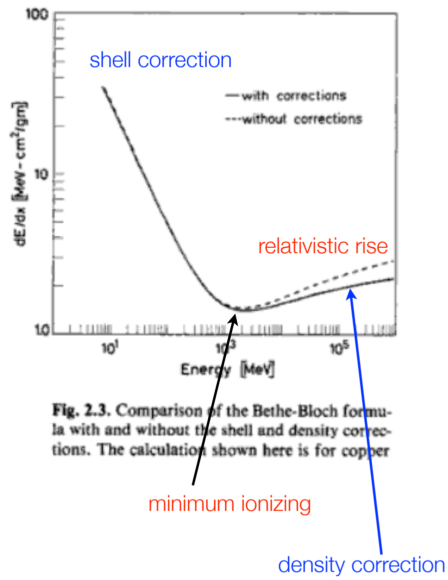
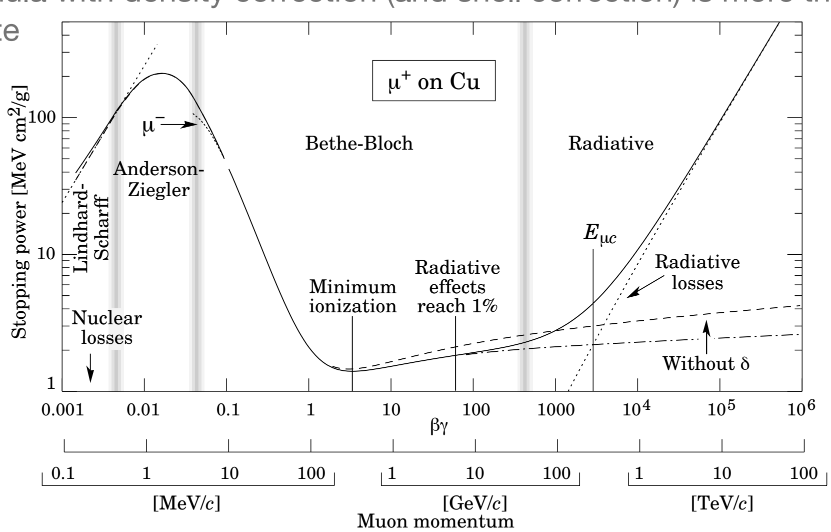


Fig. 2.3. Comparison of the Bethe-Bloch formula with and without the shell and density corrections. The calculation shown here is for copper

The Full Monty

- even within the Bethe-Bloch range, other higher order corrections (beyond the Born approximation, spin, radiative, capture of electrons) can be included but are for the most part much smaller than 1%
- the formula with density correction (and shell correction) is more than adequate



dE/dx for Mixtures and Compounds

- the Bethe-Bloch formula was presented for a single element (Z, A)
- for mixtures, a good approximation is to compute the average dE/dx weighted by the fraction of atoms (electrons) in each element

$$\frac{1}{\rho}\frac{dE}{dx} = \frac{w_1}{\rho_1}\left(\frac{dE}{dx}\right)_1 + \frac{w_2}{\rho_2}\left(\frac{dE}{dx}\right)_2 + \ldots$$

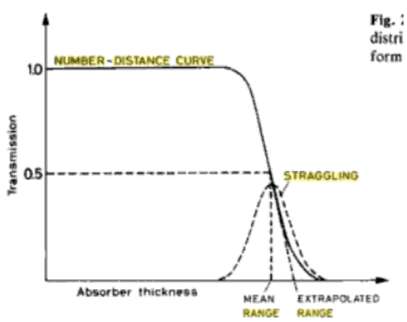
- where w_1, w_2, etc., are the weight fractions of each element
- for compounds, find Z_{eff} and A_{eff} for the molecule (e.g. CO_2)
  - Z_{eff} is the "total" Z for the molecule (for CO_2 it would be 22)
  - A_{eff} is the "total" A for the molecule (for CO_2 it would be 44)

note: Z/A determines the number of electrons and the Z doesn't affect the Coulomb interaction

- average ionization weighted by fraction of electrons $\ln I_{eff} = \sum \frac{a_i Z_i \ln I_i}{Z_{eff}}$

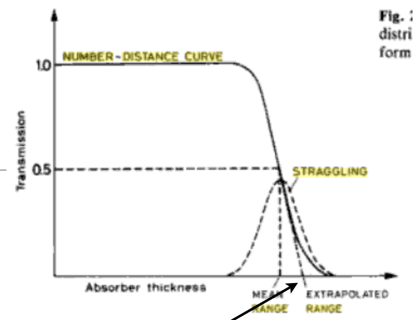
where a_i is the number of atoms of the *i*th element in the molecule

Range and Straggling



- pass a beam of particles with fixed energy
  - through different thicknesses
  - measure the fraction that are transmitted
- "straggling" is observed; energy loss is not continuous but statistical in nature and some particles undergo less/more energy loss and their range will be larger/smaller than the typical, expected range
- the variation in the amount of collisions/energy loss is approximately Gaussian (central limit theorem)
- mean range: 50% of particles transmitted
- extrapolated (or practical) range, is the thickness where you'd expect zero transmission
- continuous slowing down approximation range: $R(T_0) = \int_{T_0}^{0}\left(\frac{dE}{dx}\right)^{-1} dE$

Range and Straggling



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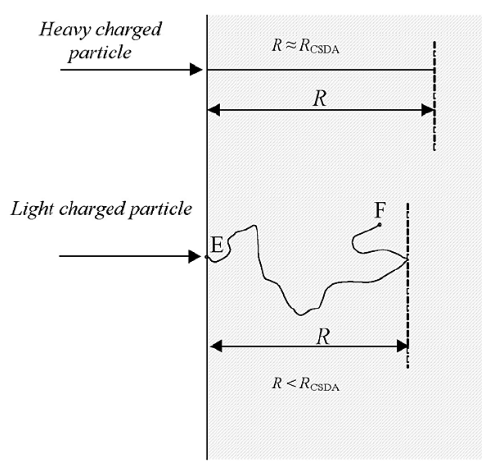
Multiple Scattering and Range

- further deviation from the theoretical range (continuous slowing down approximation) and observed comes from multiple scattering
- if the particle suffers multiple scattering, the path length it travels is longer and hence the observed range is substantially smaller than the theoretical
- for heavy charged particles, this is a small effect



Energy Loss in Thin Absorbers

- when the number of collisions/energy loss events is a large number, the central limit theorem results in the energy loss having a Gaussian distribution about the mean value
- for thin absorbers (or gases), this may not hold and the energy loss distribution has a long, high-side tail
  - probability of a large energy loss event is small, but has a big effect
- calculation of the energy loss distribution: Landau, Vavilov (also Symon)
  - Landau: applicable when (in a single collision) the mean energy loss is small compared to the maximum energy transferable

$$p(x) = \frac{1}{\pi}\int_0^\infty e^{-t\ln t - xt}\sin(\pi t)\,dt$$

  - Vavilov: applies in the region between the Landau small limit and the Gaussian limit (and approaches both at the appropriate limits)

Energy Loss in Thin Absorbers

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