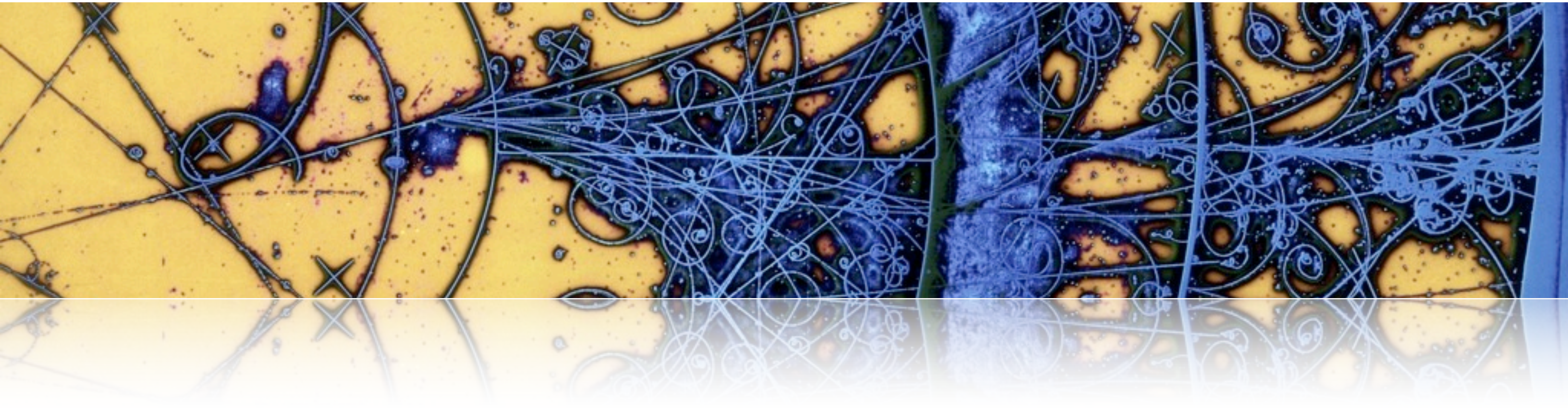
Interactions

of Particles with Matter



Particle Detection Principle

In order to detect a particle

- it must interact with the material of the detector
- transfer energy in some recognizable fashion

i.e. The detection of particles happens via their energy loss in the material it traverses ...

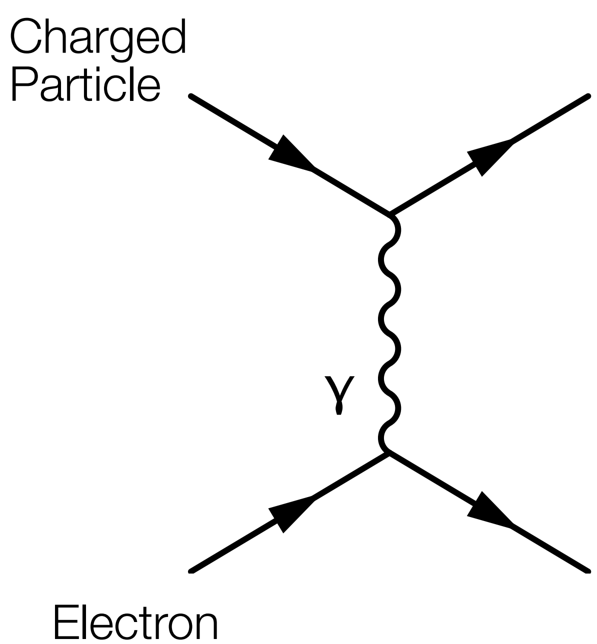
Possibilities:

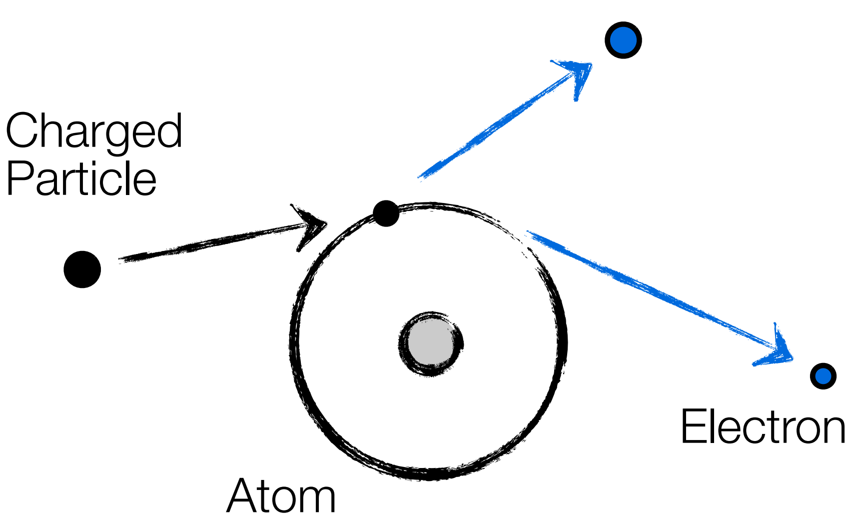
Charged particles	Ionization, Bremsstrahlung, Cherenkov ...	Energy loss by multiple reactions
Hadrons	Nuclear interactions	
Photons	Photo/Compton effect, pair production	
Neutrinos	Weak interactions	Total energy loss via single interaction

➛ charged particles

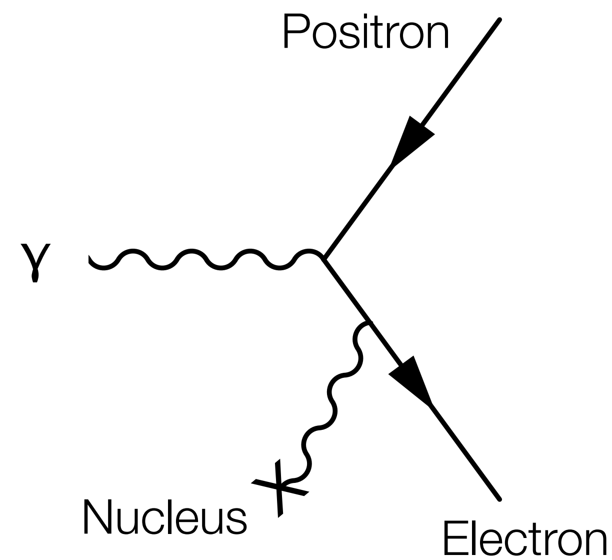
Particle Interactions – Examples

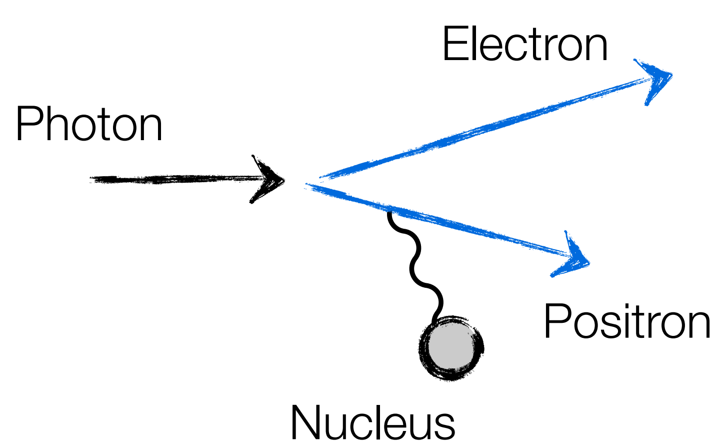
Ionization:



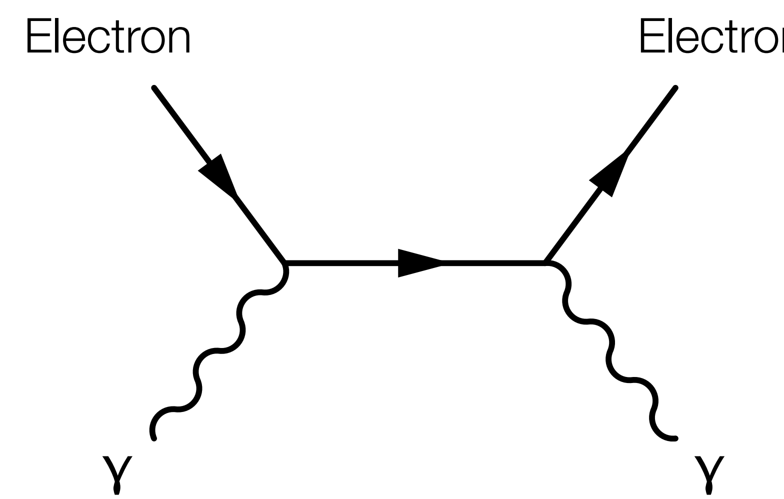


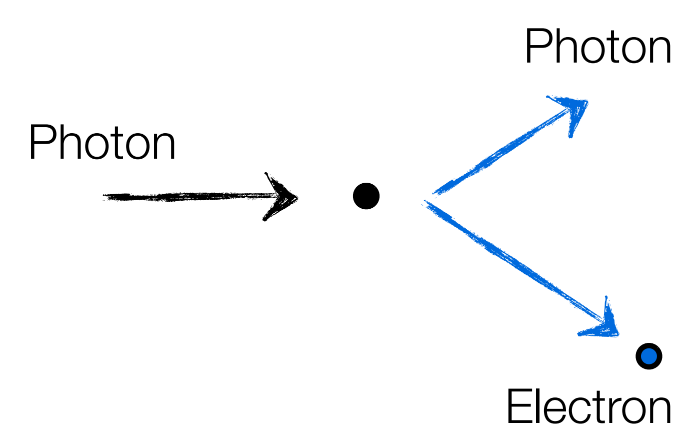
Pair production:





Compton scattering:

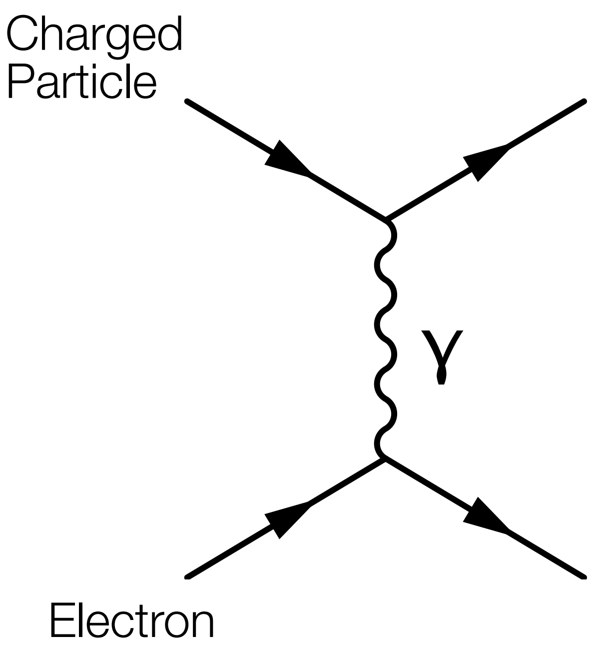




Energy Loss by Ionization – dE/dx

For now assume: $Mc^2 \gg m_e c^2$

i.e. energy loss for heavy charged particles
[dE/dx for electrons more difficult ...]



Interaction dominated
by elastic collisions with electrons ...

Bethe-Bloch Formula

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

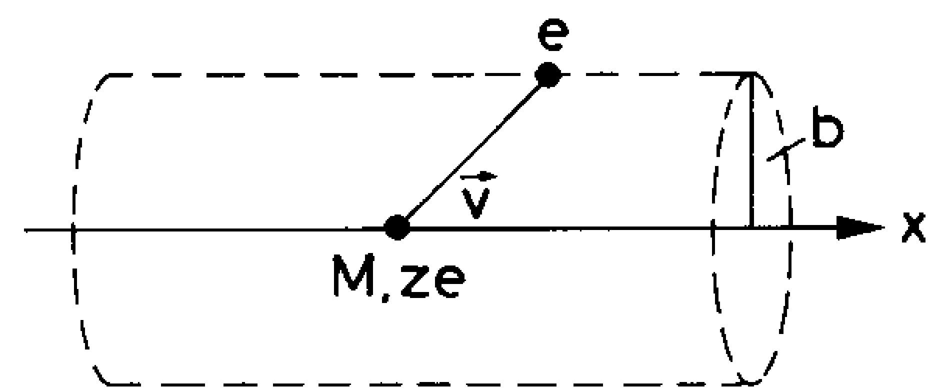
$\propto 1/\beta^2 \cdot \ln(\mathrm{const} \cdot \beta^2\gamma^2)$

Bethe-Bloch – Classical Derivation

Bohr 1913

Particle with charge ze and velocity v moves through a medium with electron density n.

Electrons considered free and initially at rest.



Interaction of a heavy charged particle with an electron of an atom inside medium.

Momentum transfer:

Symmetry!

$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dt}{dx} dx = \int F_\perp \frac{dx}{v} \qquad \Delta p_{||} \ : \ \text{averages to zero}$$

$$= \int_{-\infty}^{\infty} \frac{ze^2}{(x^2+b^2)} \cdot \frac{b}{\sqrt{x^2+b^2}} \cdot \frac{1}{v}\, dx = \frac{ze^2 b}{v} \left[\frac{x}{b^2\sqrt{x^2+b^2}} \right]_{-\infty}^{\infty} = \frac{2ze^2}{bv}$$

More elegant with Gauss law:
[infinite cylinder; electron in center]

$$\int E_\perp\,(2\pi b)\, dx = 4\pi(ze) \to \int E_\perp dx = \frac{2ze}{b}$$

and then ...

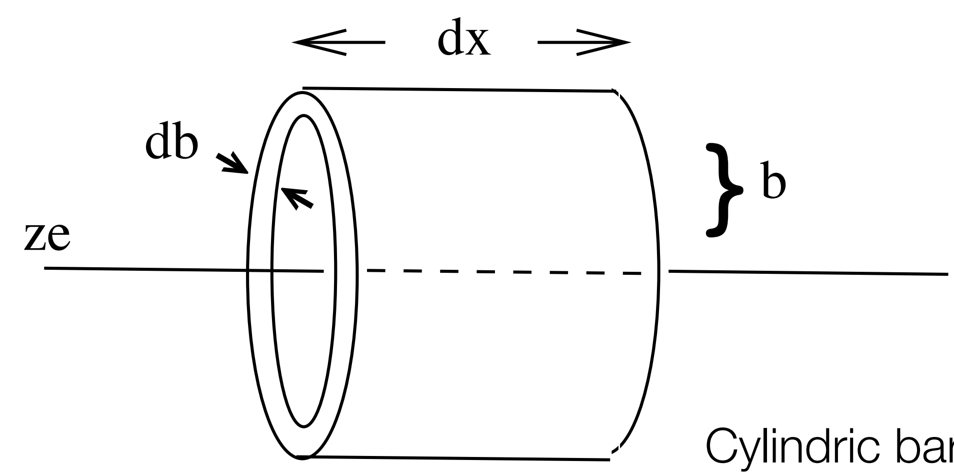
$$\begin{cases} F_\perp = eE_\perp \\ \Delta p_\perp = e\int E_\perp \frac{dx}{v} = \frac{2ze^2}{bv} \end{cases}$$

Bethe-Bloch – Classical Derivation

Bohr 1913

Energy transfer onto single electron
for impact parameter b:

$$\Delta E(b) = \frac{\Delta p^2}{2m_\mathrm{e}}$$



Cylindric barrel
with N_e electrons

Consider cylindric barrel ➛ $N_e = n \cdot (2\pi b) \cdot db\, dx$

Energy loss per path length dx for
distance between b and b+db in medium with electron density n:

Energy loss!

$$-dE(b) = \frac{\Delta p^2}{2m_\mathrm{e}} \cdot 2\pi n b\, db\, dx = \frac{4z^2e^4}{2b^2v^2m_\mathrm{e}} \cdot 2\pi n b\, db\, dx = \frac{4\pi\, n\, z^2e^4}{m_\mathrm{e}v^2}\frac{db}{b}dx$$

Diverges for b ➛ 0; integration only
for relevant range [b_{min}, b_{max}]:

Bohr 1913

$$-\frac{dE}{dx} = \frac{4\pi\, n\, z^2e^4}{m_\mathrm{e}v^2} \cdot \int_{b_\mathrm{min}}^{b_\mathrm{max}} \frac{db}{b} = \frac{4\pi\, n\, z^2e^4}{m_\mathrm{e}v^2} \ln\frac{b_\mathrm{max}}{b_\mathrm{min}}$$

Bethe-Bloch – Classical Derivation

Bohr 1913

Determination of relevant range [b_{min}, b_{max}]:
[Arguments: $b_{min} > \lambda_e$, i.e. de Broglie wavelength; $b_{max} < \infty$ due to screening ...]

$$b_{\mathrm{min}} = \lambda_{\mathrm{e}} = \frac{h}{p} = \frac{2\pi\hbar}{\gamma m_{\mathrm{e}} v}$$

Use Heisenberg uncertainty principle or that electron is located within de Broglie wavelength ...

$$b_{\mathrm{max}} = \frac{\gamma v}{\langle \nu_{\mathrm{e}} \rangle} \; ; \quad \left[\; \gamma = \frac{1}{\sqrt{1-\beta^2}} \; \right]$$

Interaction time (b/v) must be much shorter than period of the electron (γ/ν_e) to guarantee relevant energy transfer ...

[adiabatic invariance]

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_{\mathrm{e}}\, c^2 \beta^2}\, n \cdot \ln \frac{m_{\mathrm{e}}\, c^2 \beta^2 \gamma^2}{2\pi\hbar \, \langle \nu_{\mathrm{e}} \rangle}$$

Deviates by factor 2 from QM derivation

Electron density: $n = N_A \cdot \rho \cdot Z/A$!!

Effective Ionization potential: $I \sim h \langle \nu_e \rangle$

Bethe-Bloch Formula

[see e.g. PDG 2010]

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] \quad [\cdot \rho]$$

density

$K = 4\pi N_A r_e^2 m_e c^2 = 0.307$ MeV g^{-1} cm^2

$T_{max} = 2 m_e c^2 \beta^2 \gamma^2 / (1 + 2\gamma\, m_e/M + (m_e/M)^2)$

[Max. energy transfer in single collision]

z : Charge of incident particle
M : Mass of incident particle

Z : Charge number of medium
A : Atomic mass of medium

I : Mean excitation energy of medium
δ : Density correction [transv. extension of electric field]

$N_A = 6.022 \cdot 10^{23}$
[Avogardo's number]

$r_e = e^2/4\pi\varepsilon_0 m_e c^2 = 2.8$ fm
[Classical electron radius]

$m_e = 511$ keV
[Electron mass]

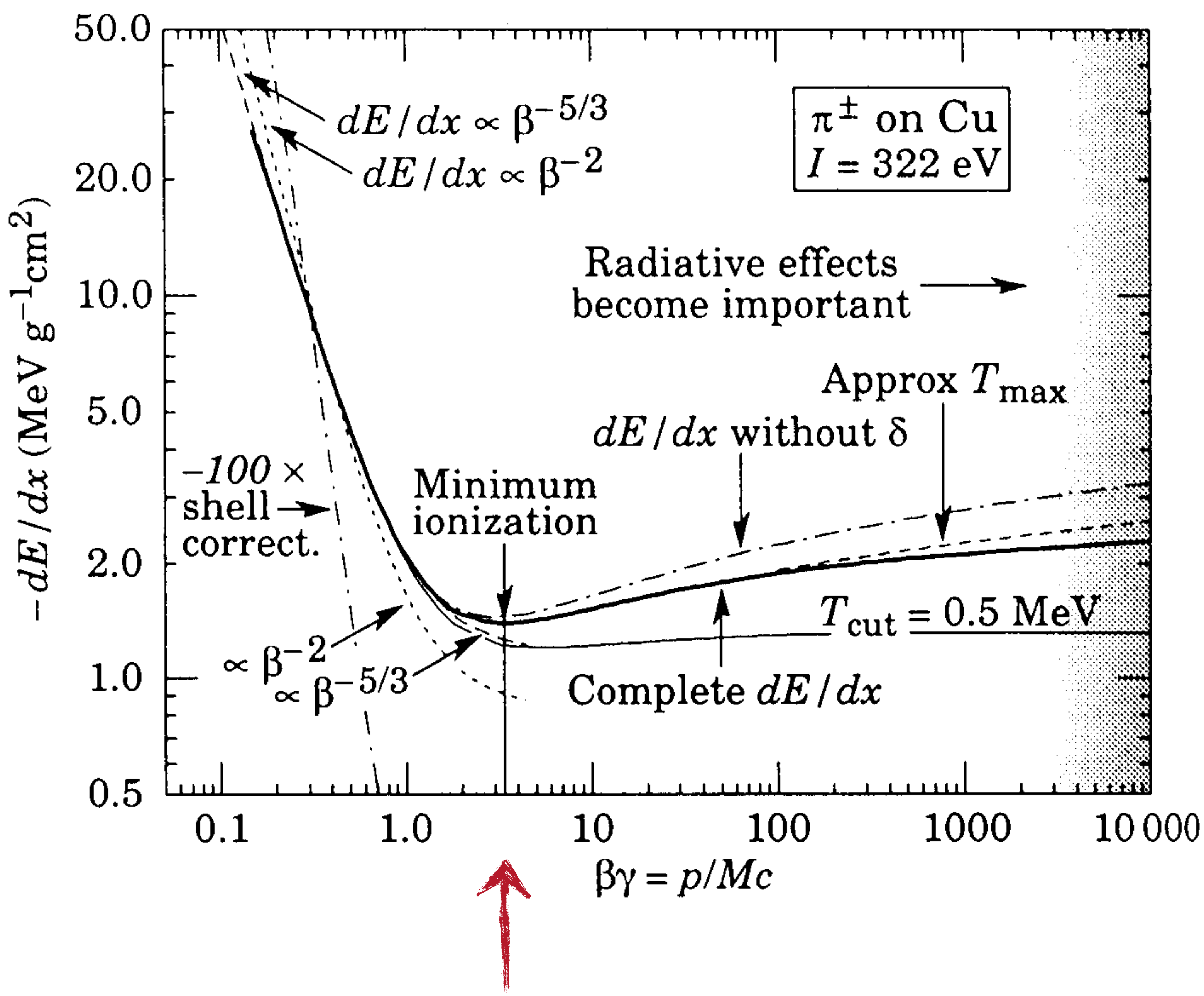
$\beta = v/c$
[Velocity]

$\gamma = (1-\beta^2)^{-2}$
[Lorentz factor]

Validity:

$.05 < \beta\gamma < 500$
$M > m_\mu$

Energy Loss of Pions in Cu



$\beta\gamma$ = 3 - 4

Minimum ionizing particles (MIP): $\beta\gamma$ = 3-4

dE/dx falls ~ β^{-2}; kinematic factor
[precise dependence: ~ $\beta^{-5/3}$]

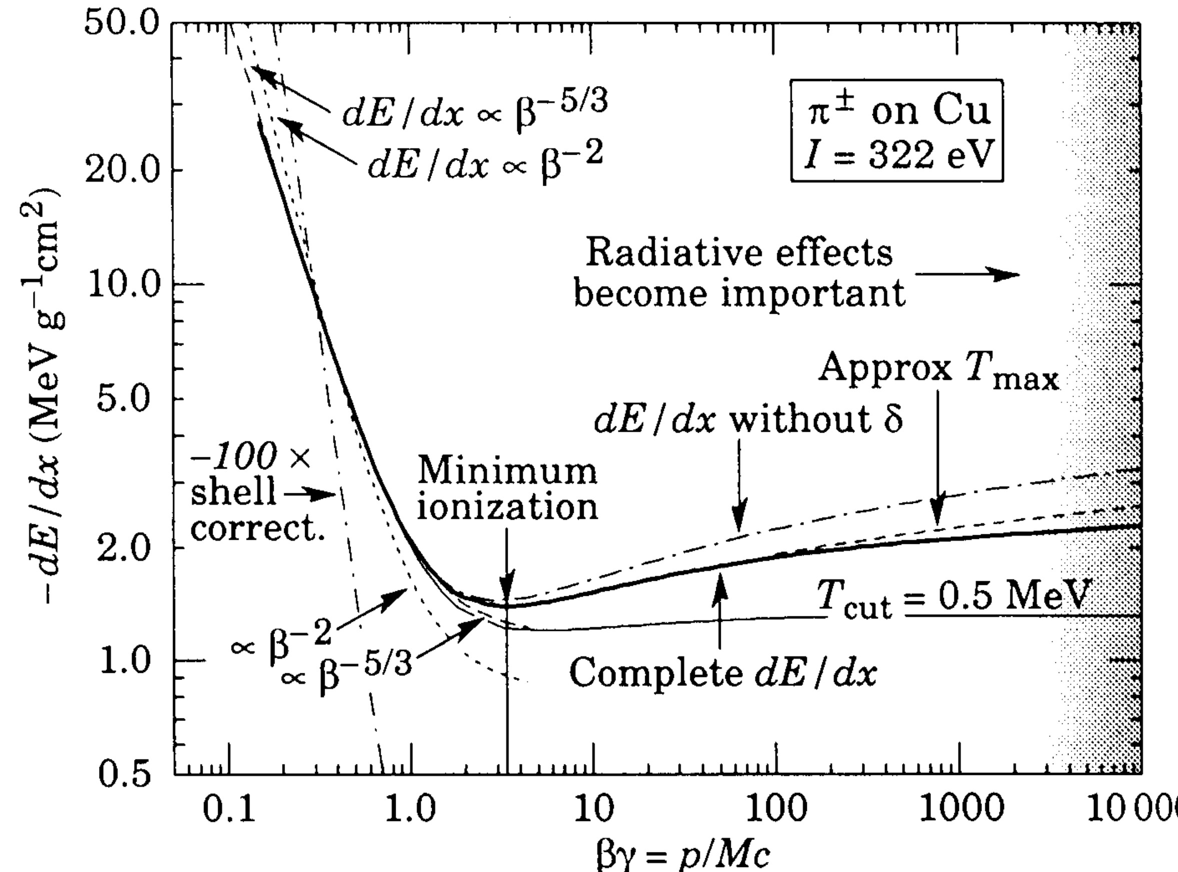
dE/dx rises ~ $\ln(\beta\gamma)^2$; relativistic rise
[rel. extension of transversal E-field]

Saturation at large ($\beta\gamma$) due to density effect (correction δ)
[polarization of medium]

Units: MeV g^{-1} cm^2

MIP looses ~ 13 MeV/cm
[density of copper: 8.94 g/cm^3]

Understanding Bethe-Bloch



$1/\beta^2$-dependence:

Remember:

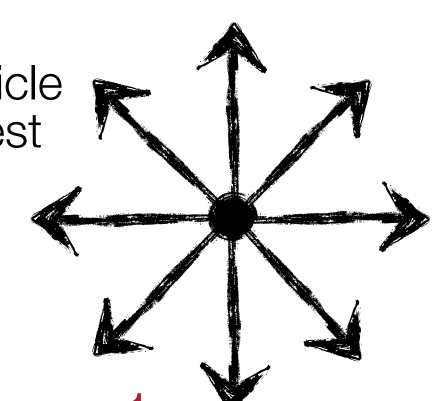
$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dx}{v}$$

i.e. slower particles feel electric force of atomic electron for longer time ...

Relativistic rise for $\beta\gamma > 4$:

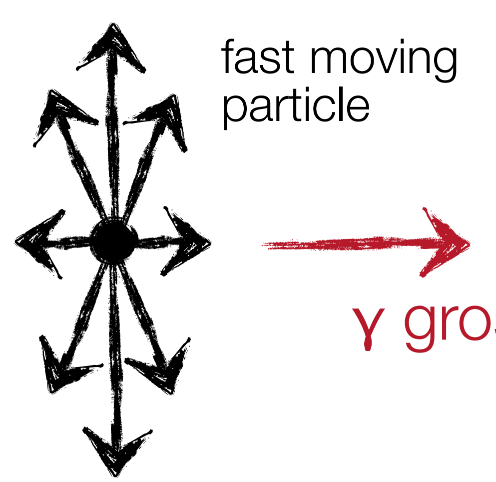
High energy particle: transversal electric field increases due to Lorentz transform; $E_y \rightarrow \gamma E_y$. Thus interaction cross section increases ...

particle at rest



$\gamma = 1$

fast moving particle



γ gross

Corrections:

low energy : shell corrections

high energy : density corrections

Understanding Bethe-Bloch

Density correction:

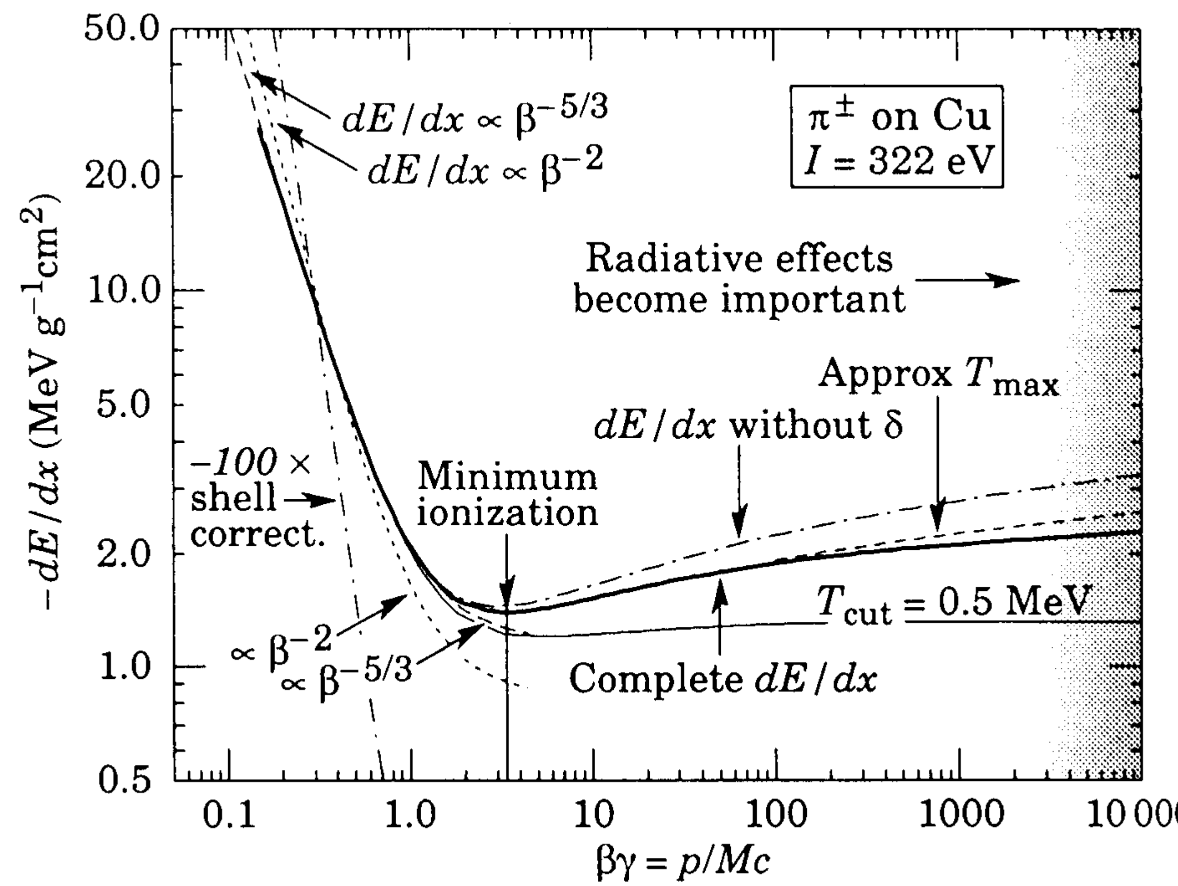
Polarization effect ...
[density dependent]

➛ Shielding of electrical field far from particle path; effectively cuts of the long range contribution ...

More relevant at high γ ...
[Increased range of electric field; larger b_{max}; ...]

For high energies:

$$\delta/2 \rightarrow \ln(\hbar\omega/I) + \ln \beta\gamma - 1/2$$



Density effect leads to saturation at high energy ...

Shell correction:

Arises if particle velocity is close to orbital velocity of electrons, i.e. $\beta c \sim v_e$.

Assumption that electron is at rest breaks down ...
Capture process is possible ...

Shell correction are in general small ...

Energy Loss of Charged Particles

Dependence on

Mass A

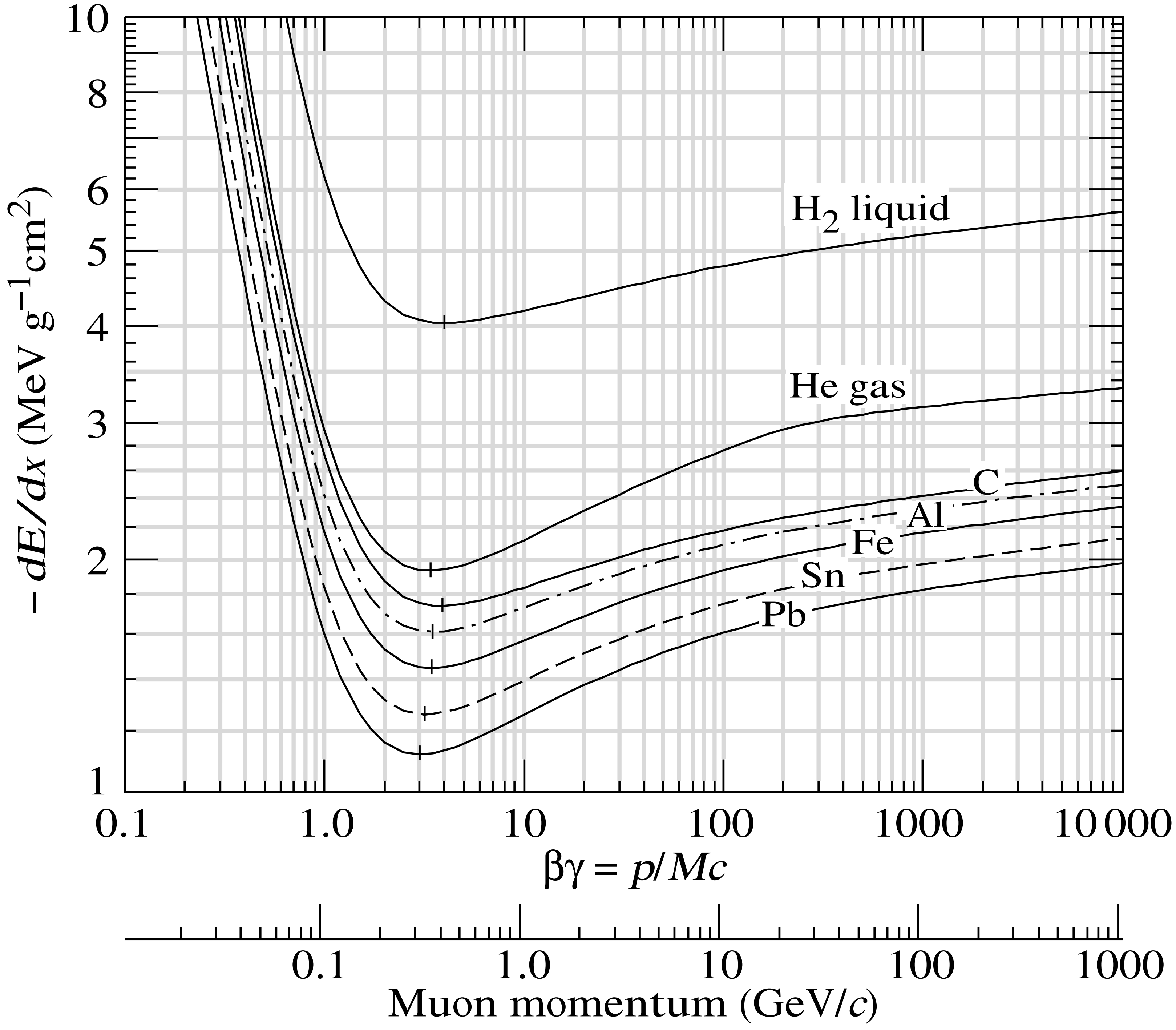
Charge Z

of target nucleus

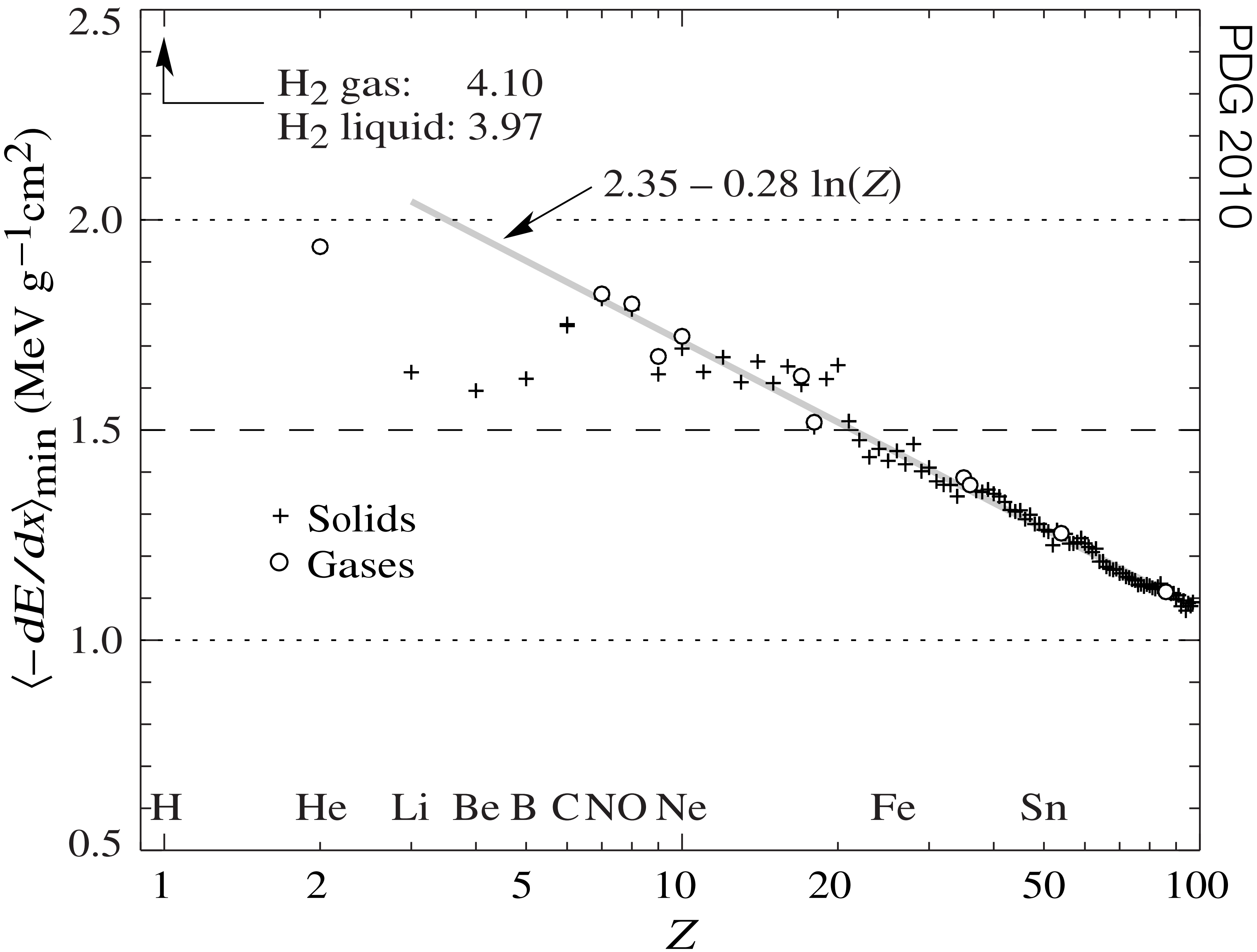
Minimum ionization:

ca. 1 - 2 MeV/g cm^{-2}

[H_2: 4 MeV/g cm^{-2}]

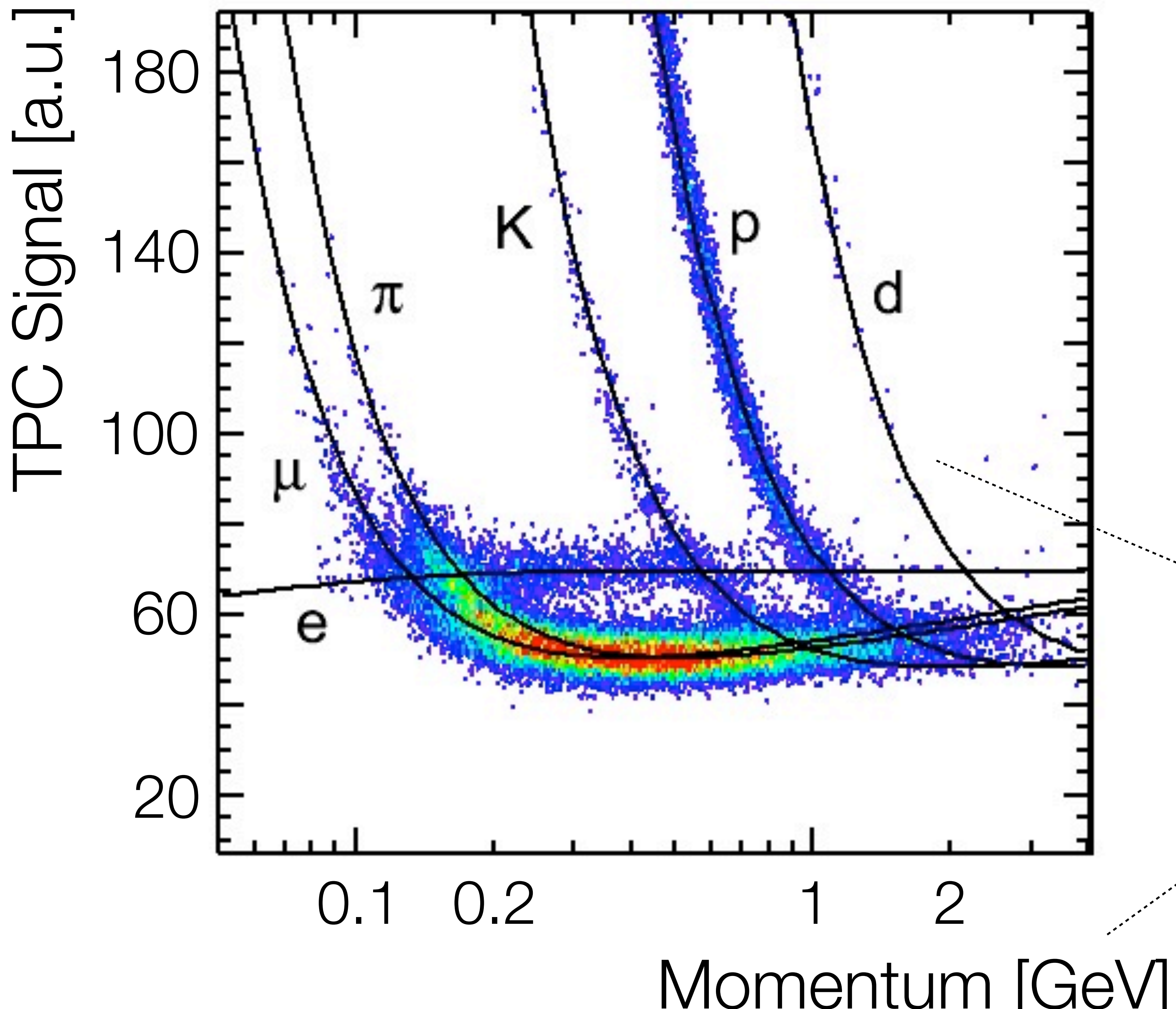


Stopping Power at Minimum Ionization



Stopping power at minimum ionization for the chemical elements. The straight line is fitted for Z > 6.
A simple functional dependence on Z is not to be expected, since <–dE/dx> also depends on other variables.

dE/dx and Particle Identification



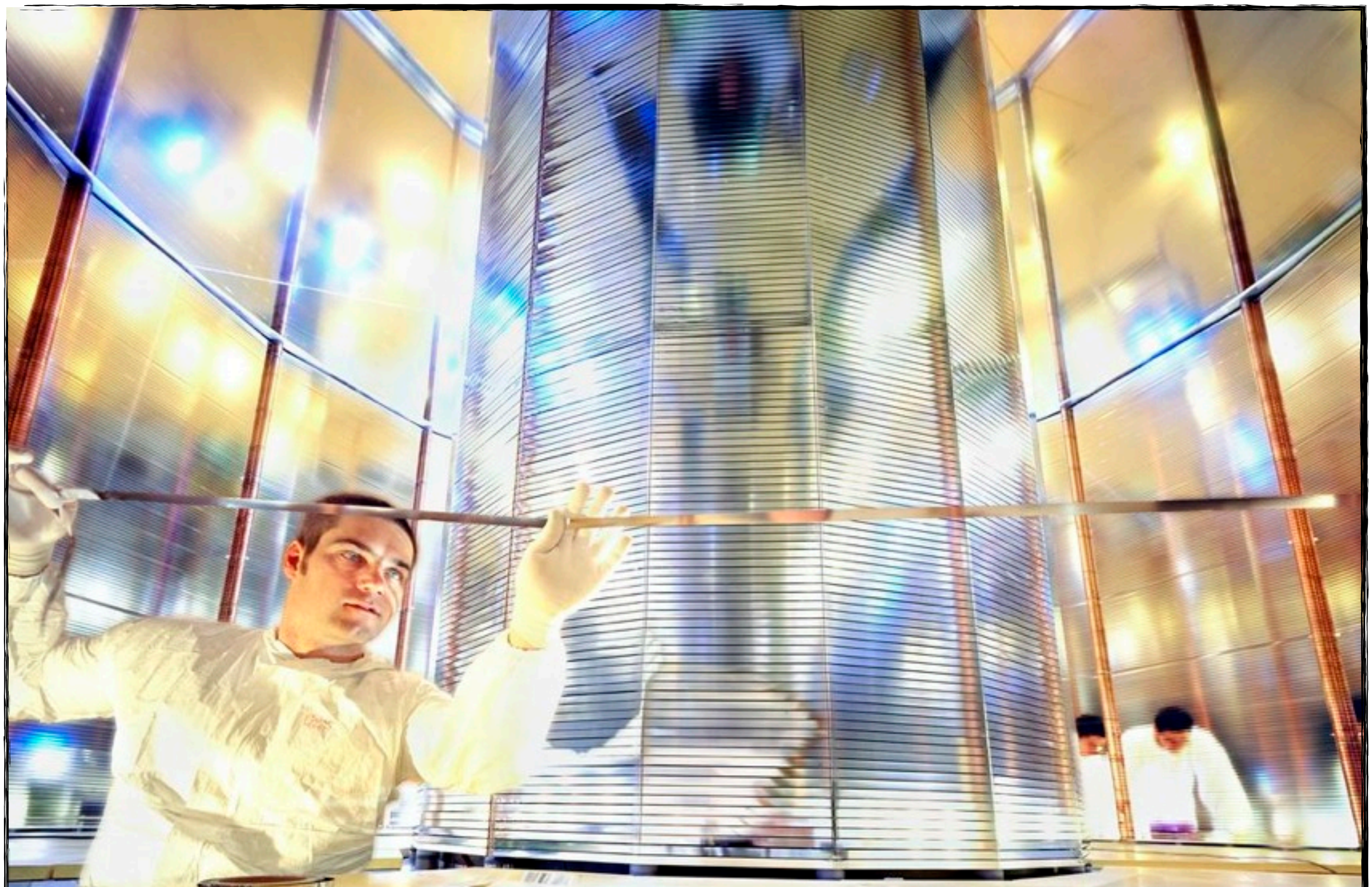
Measured
energy loss

[ALICE TPC, 2009]

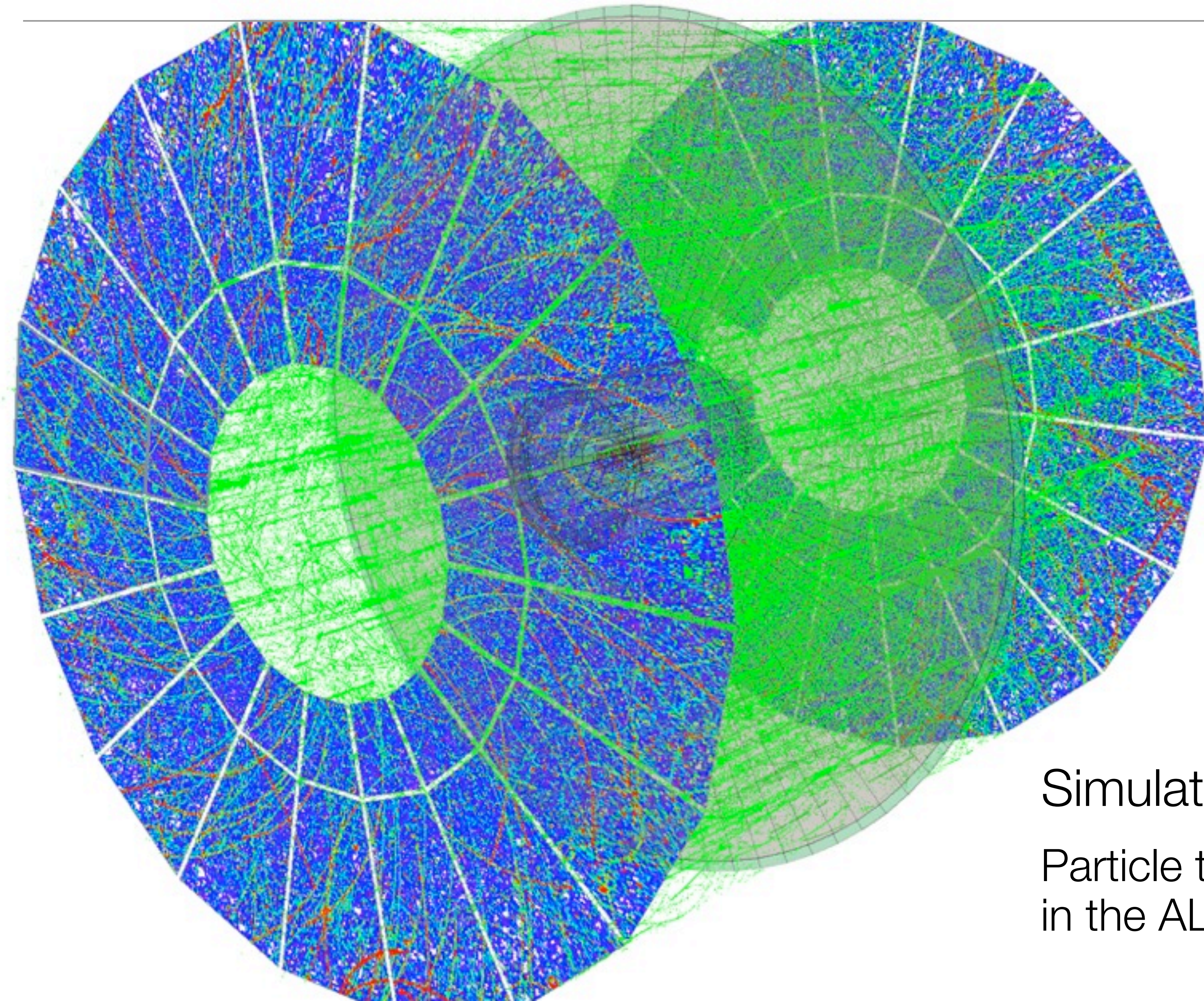
Bethe-Bloch

Remember:
dE/dx depends on β!

The ALICE TPC



The ALICE TPC



Simulation

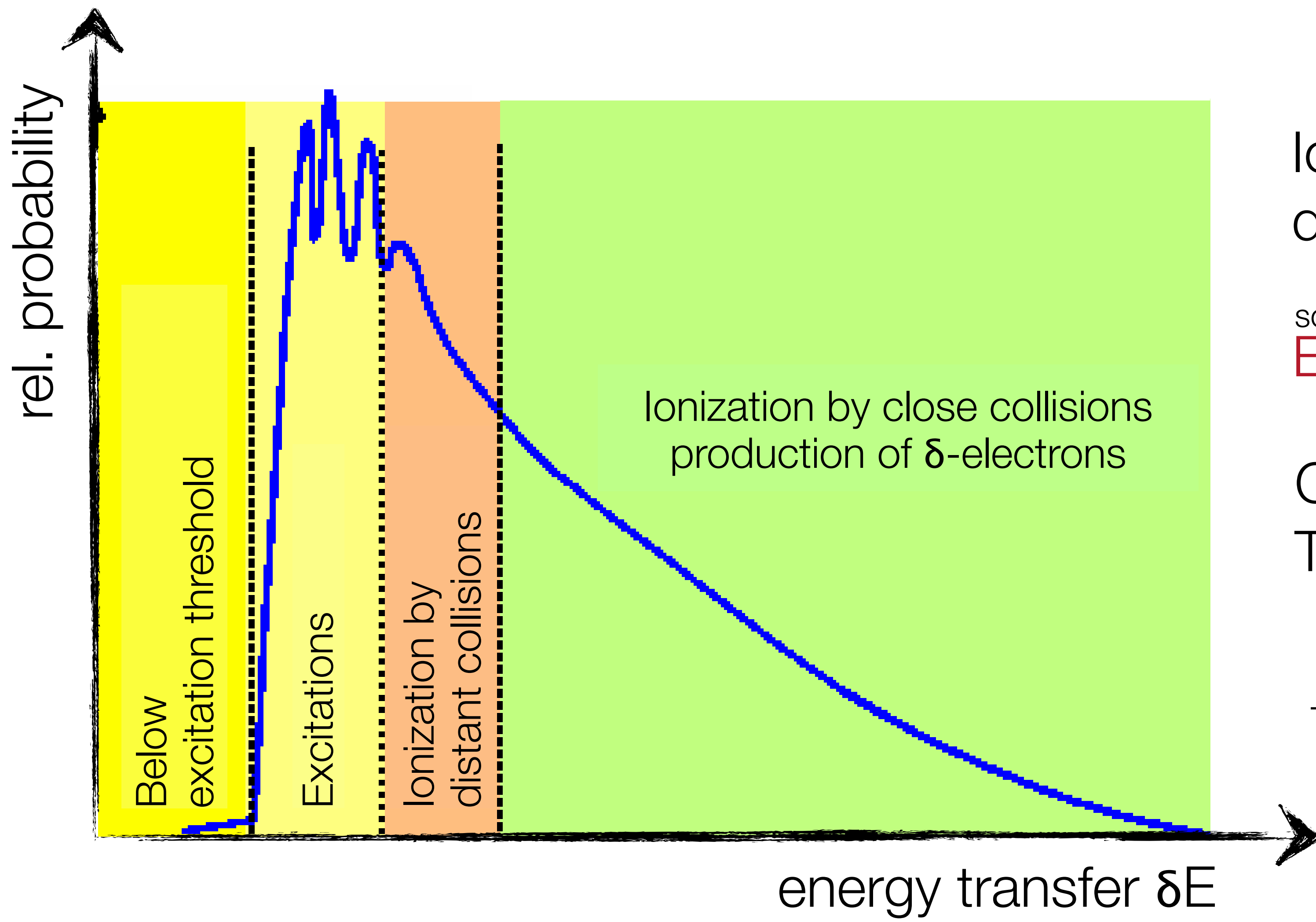
Particle tracks
in the ALICE TPC

dE/dx Fluctuations

Bethe-Bloch describes mean energy loss; measurement via energy loss ΔE in a material of thickness Δx with

$$\Delta E = \sum_{n=1}^{N} \delta E_n$$

N : number of collisions
δE : energy loss in a single collision



Ionization loss δE distributed statistically ...

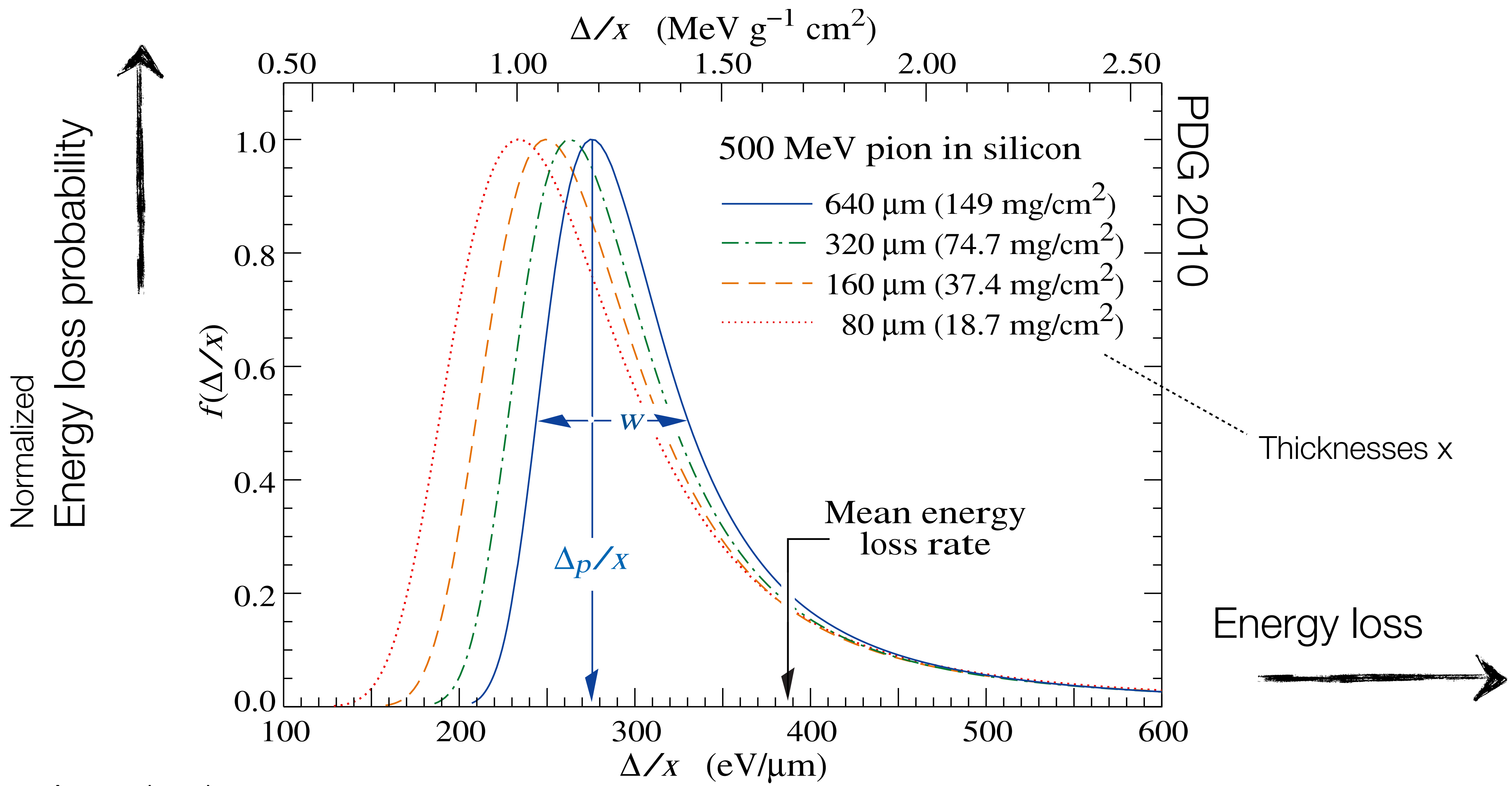
so-called
Energy loss 'straggling'

Complicated problem ...
Thin absorbers: Landau distribution

Standard Gauss with mean energy loss E_0
\+ tail towards high energies due to δ-electrons

see also Allison & Cobb
[Ann. Rev. Nucl. Part. Sci. 30 (1980) 253.]

dE/dx Fluctuations – Landau Distribution



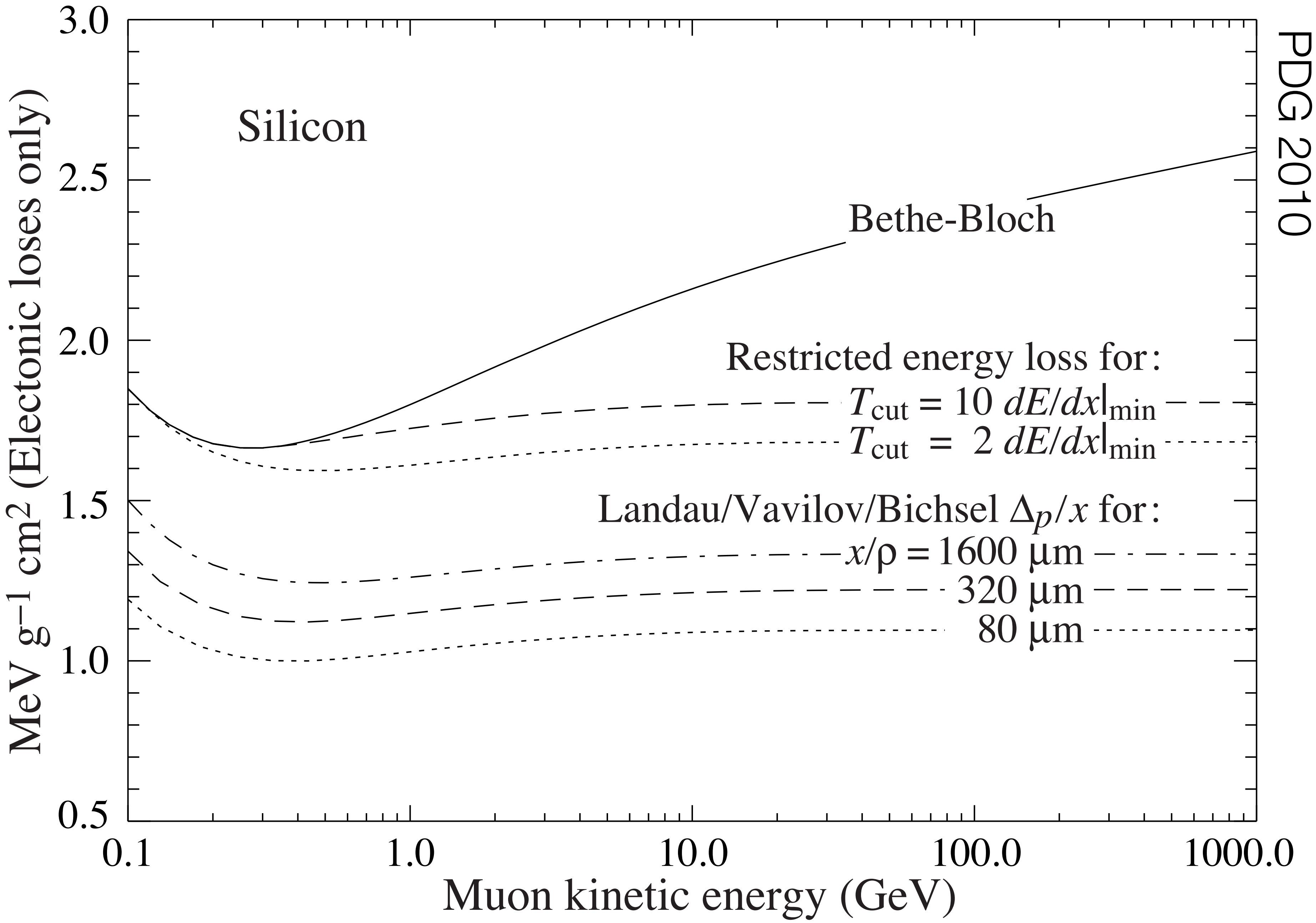
Approximation:

$$f(\Delta/x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\Delta/x - a(\Delta/x)_{\text{mip}}}{\xi} \right) + e^{-\left(\frac{\Delta/x - a(\Delta/x)_{\text{mip}}}{\xi} \right)} \right]$$

for full form see e.g. Leo

ξ: material constant

dE/dx Fluctuations – Landau Distribution



Bethe-Bloch dE/dx, two examples of restricted energy loss, and the Landau most probable energy per unit thickness in silicon. The change of Δ_p/x with thickness x illustrates its a ln x + b dependence. Minimum ionization ($dE/dx|_{min}$) is 1.664 MeV g^{-1} cm^2. Radiative losses are excluded. The incident particles are muons.

Energy Loss at Small Energies

Shell corrections

to correct for atomic binding

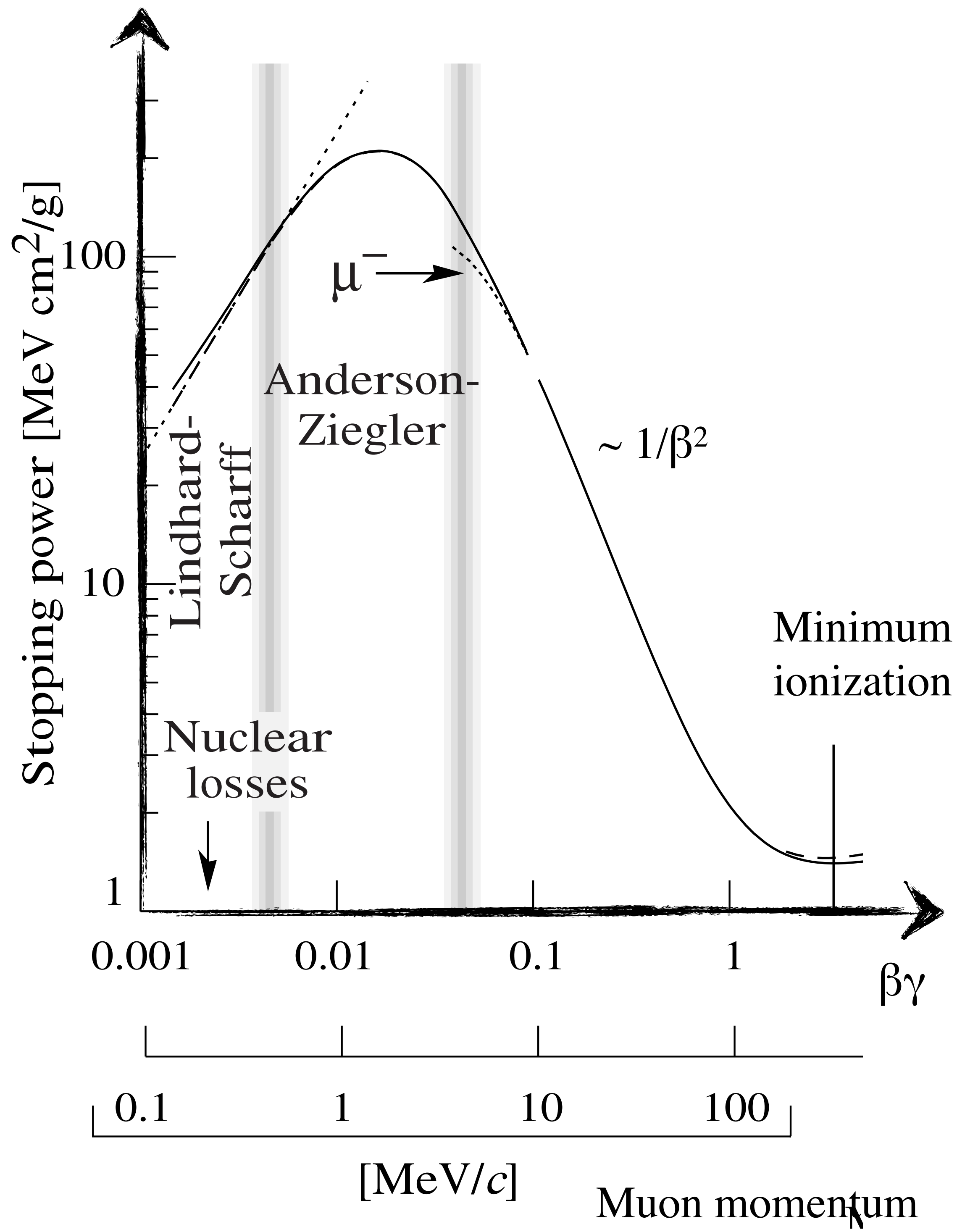
Higher order corrections

relevant only at low energies

Bloch corrections (higher orders)
Barkas correction (pos. vs. neg. charge)

With these corrections
BB yields 1% accuracy
down to $\beta\gamma$ =0.05.

For $\beta\gamma$ < 0.05 there are only
phenomenological fitting formulae available.



Mean Particle Range

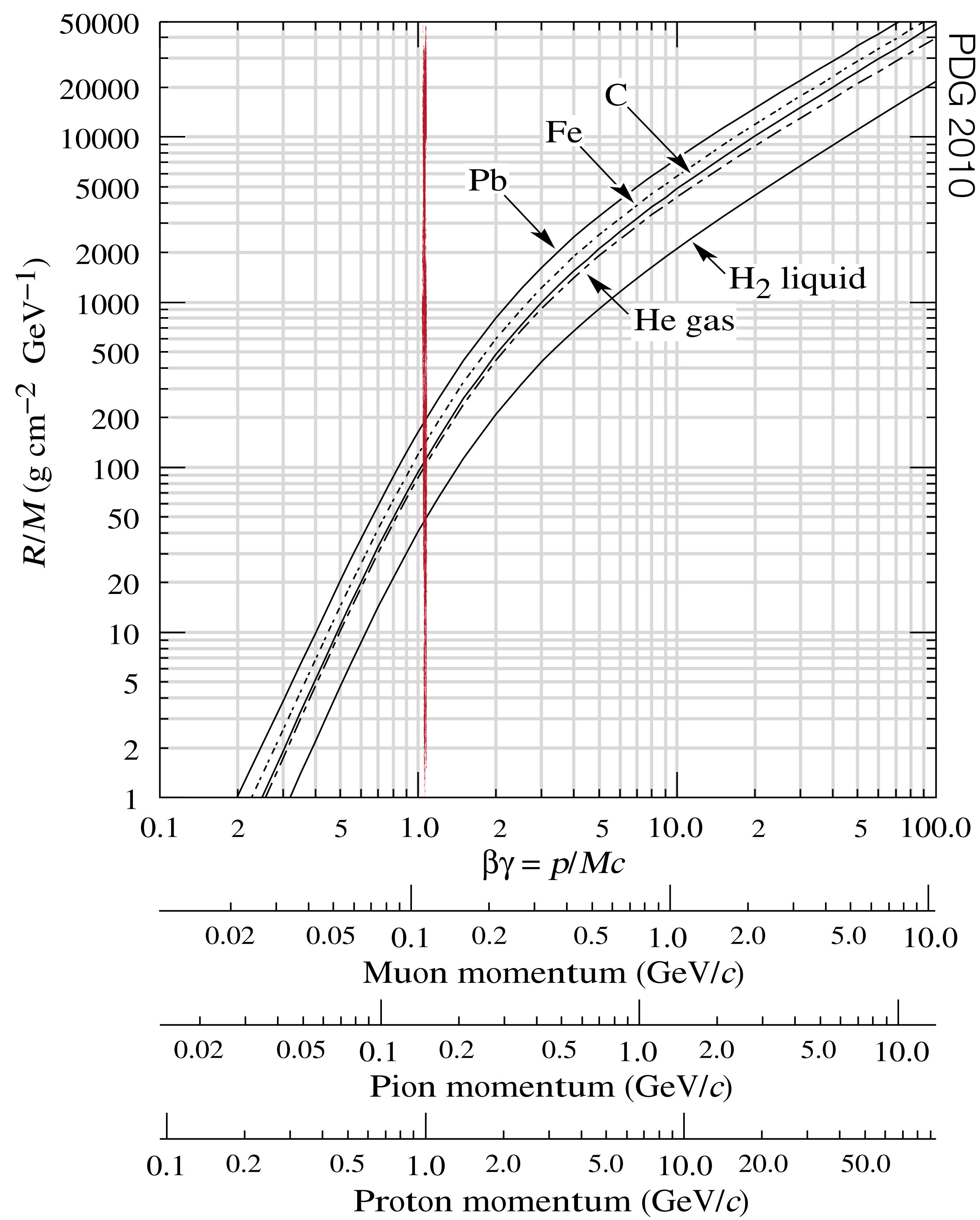
Integrate over energy loss
from E down to 0

$$R = \int_E^0 \frac{dE}{dE/dx}$$

Example:

Proton with p = 1 GeV
Target: lead with **ρ** = 11.34 g/cm^3

R/M = 200 g cm^{-2} GeV^{-1}
➞ R = 200/11.34/1 cm ~ 20 cm



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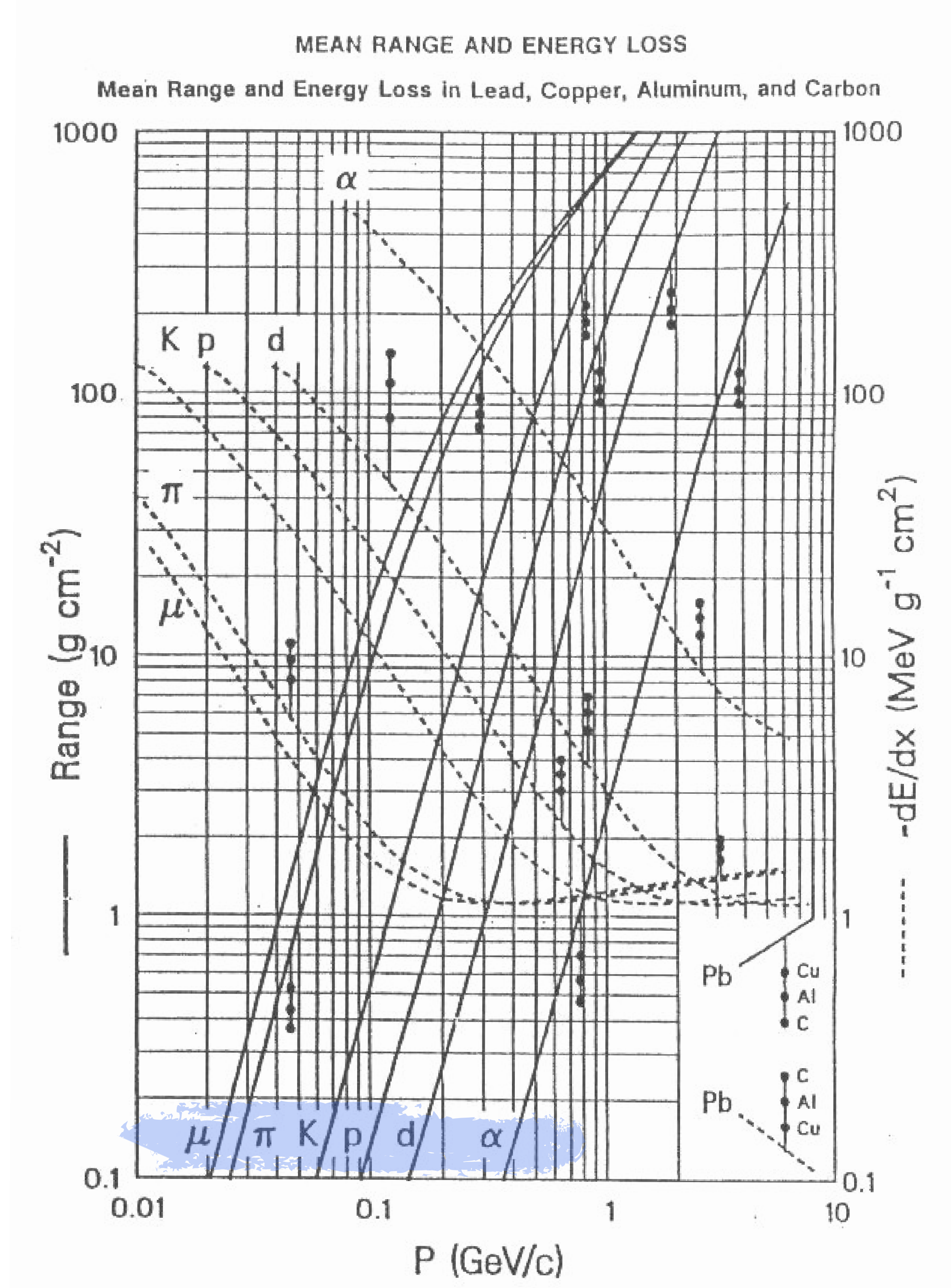


Particle Energy Deposit

βγ > 3.5:

$$\left\langle \frac{dE}{dx} \right\rangle \approx \left.\frac{dE}{dx}\right|_{\min}$$

βγ < 3.5:

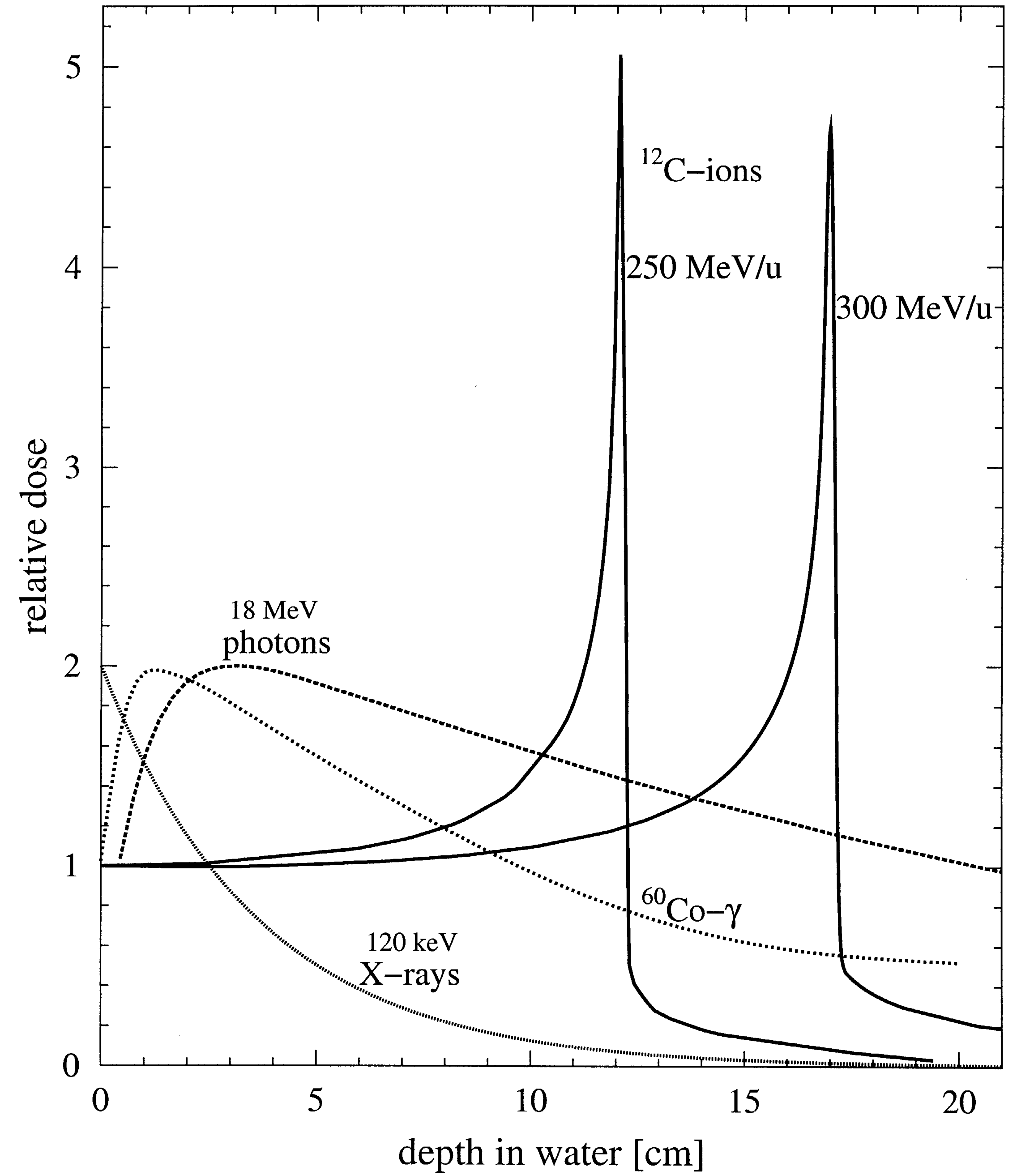
$$\left\langle \frac{dE}{dx} \right\rangle \gg \left.\frac{dE}{dx}\right|_{\min}$$

Applications:

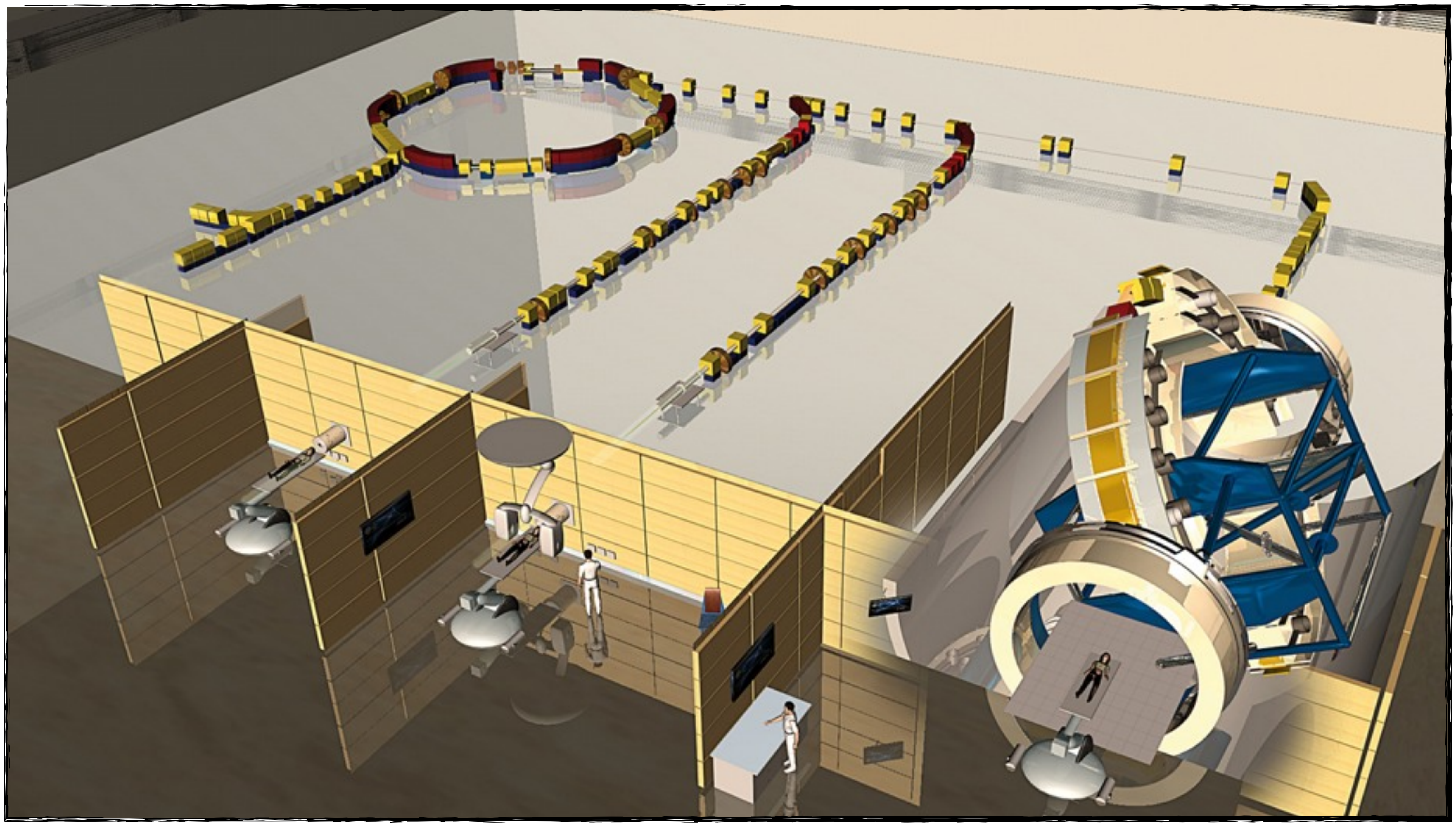
Tumor therapy

Possibility to precisely deposit dose at well defined depth by E_{beam} variation

[see Journal Club]



Heidelberg Ion-Beam Therapy Center (HIT)



Energy Loss of Electrons

Bethe-Bloch formula needs modification

Incident and target electron have same mass m_e
Scattering of identical, undistinguishable particles

$$-\left\langle \frac{dE}{dx} \right\rangle_{\text{el.}} = K \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{m_e \beta^2 c^2 \gamma^2 T}{2I^2} + F(\gamma) \right]$$

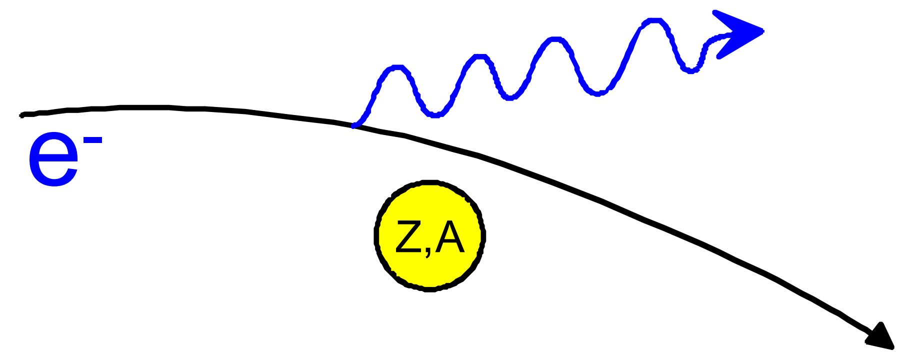
[T: kinetic energy of electron]

$W_{max} = ½T$

Remark: different energy loss for electrons and positrons at low energy as positrons are not identical with electrons; different treatment ...

Bremsstrahlung

Bremsstrahlung arises if particles
are accelerated in Coulomb field of nucleus



$$\frac{dE}{dx} = 4\alpha N_A \, \frac{z^2 Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \, \ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

i.e. energy loss proportional to $1/m^2$ ➛ main relevance for electrons ...

... or ultra-relativistic muons

Consider electrons:

$$\frac{dE}{dx} = 4\alpha N_A \, \frac{Z^2}{A} r_e^2 \cdot E \, \ln \frac{183}{Z^{\frac{1}{3}}}$$

$$\frac{dE}{dx} = \frac{E}{X_0} \qquad \text{with} \quad X_0 = \frac{A}{4\alpha N_A \, Z^2 r_e^2 \, \ln \frac{183}{Z^{\frac{1}{3}}}}$$

[Radiation length in g/cm^2]

➡ $E = E_0 e^{-x/X_0}$

After passage of one X_0 electron has lost all but $(1/e)^{th}$ of its energy

[i.e. 63%]

Bremsstrahlung – Critical Energy

Critical energy:

$$\frac{dE}{dx}(E_c)\Big|_{\mathrm{Brems}} = \frac{dE}{dx}(E_c)\Big|_{\mathrm{Ion}}$$

Approximation:

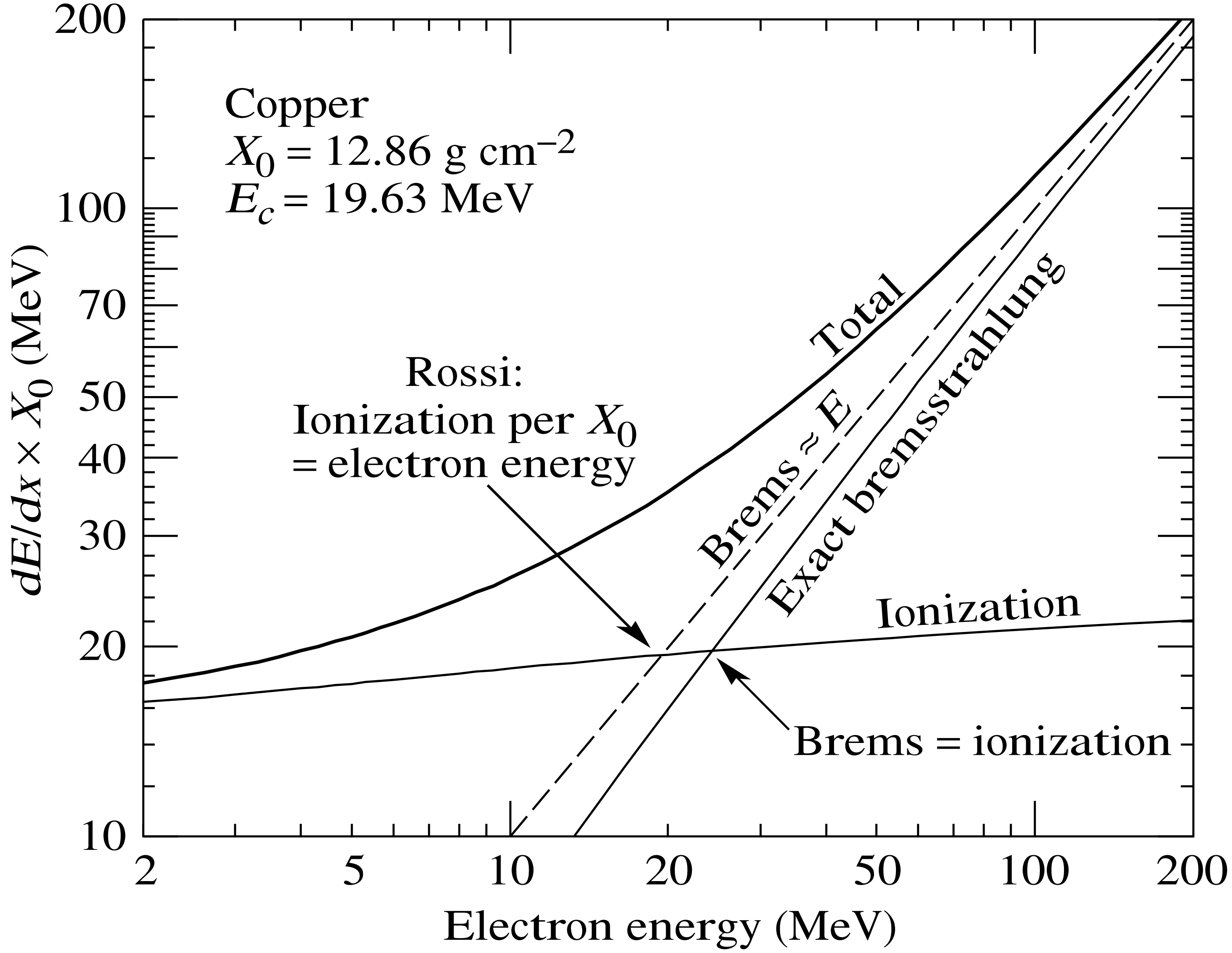
$$E_c^{\mathrm{Gas}} = \frac{710\ \mathrm{MeV}}{Z+0.92}$$

$$E_c^{\mathrm{Sol/Liq}} = \frac{610\ \mathrm{MeV}}{Z+1.24}$$

Example Copper:

$E_c \approx 610/30$ MeV ≈ 20 MeV

$$\left(\frac{dE}{dx}\right)_{\mathrm{Tot}} = \left(\frac{dE}{dx}\right)_{\mathrm{Ion}} + \left(\frac{dE}{dx}\right)_{\mathrm{Brems}}$$

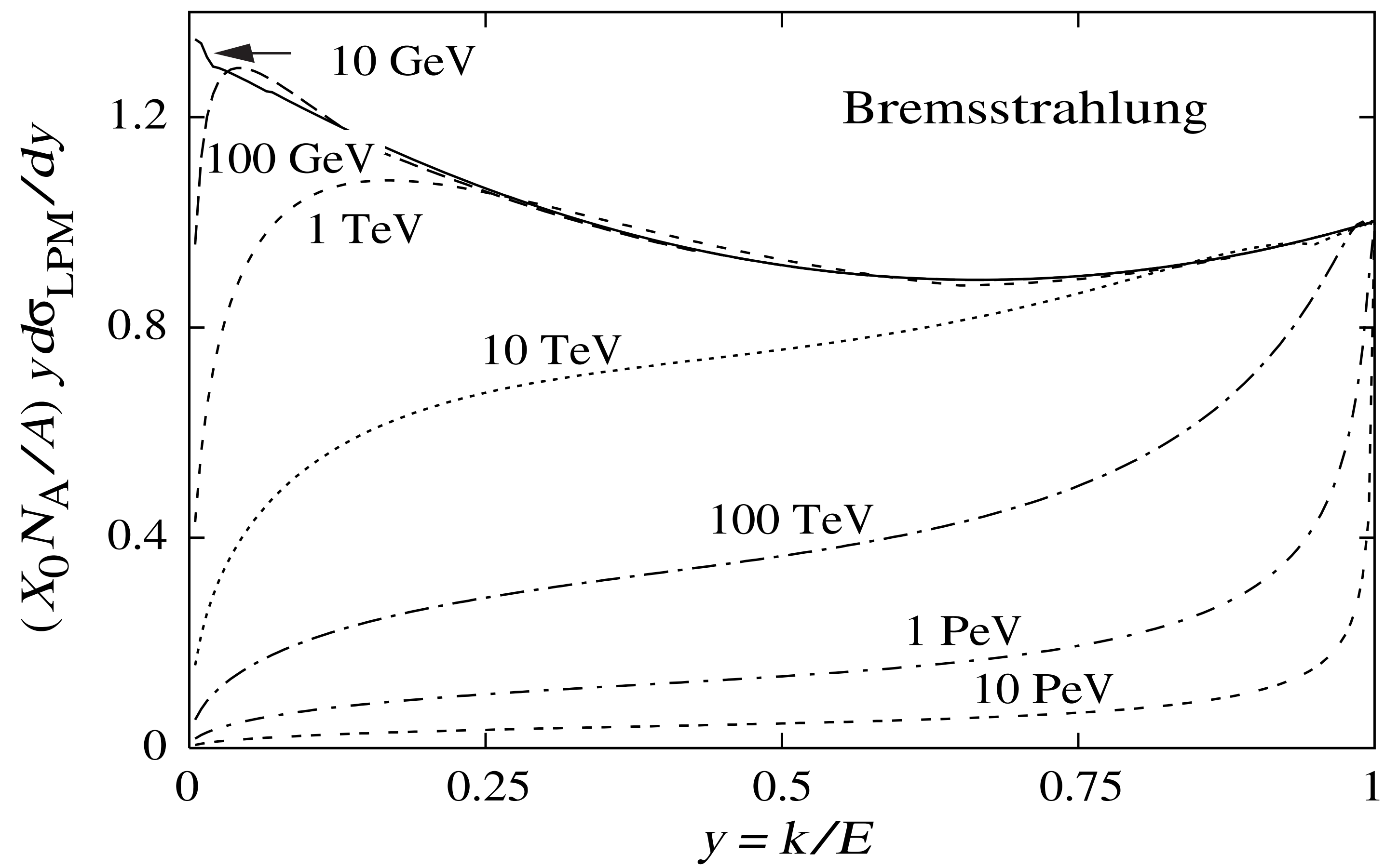


Bremsstrahlung – Energy Spectrum

Cross Section
[high energy approximation]

$$\frac{d\sigma}{dk} = \frac{1}{k}\frac{A}{X_0 N_A}\left(\frac{4}{3} - \frac{4}{3}y + y^2\right)$$

[Main dependence: dσ/dk ~ 1/k]



Formula
accurate for a large y range ...

except near y = 1 and near y = 0;
deviations greater for higher electron energies.
[see PDG for further details]

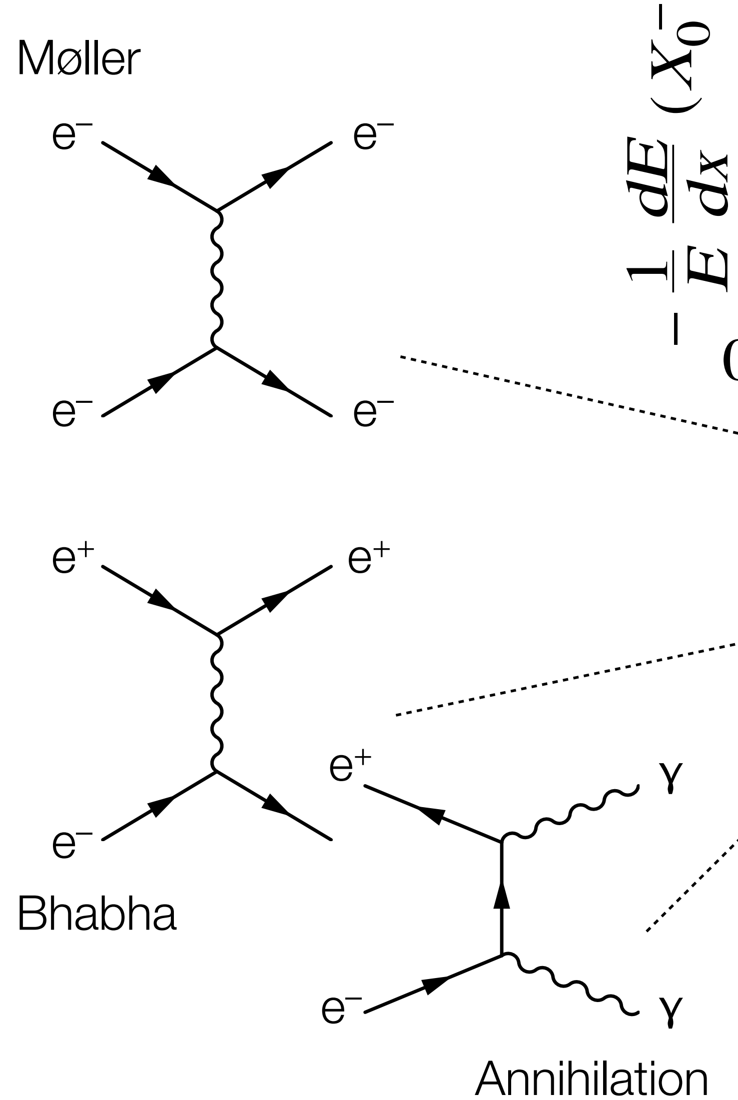
Normalized Bremsstrahlung cross section
k dσ/dk in lead versus fractional photon energy

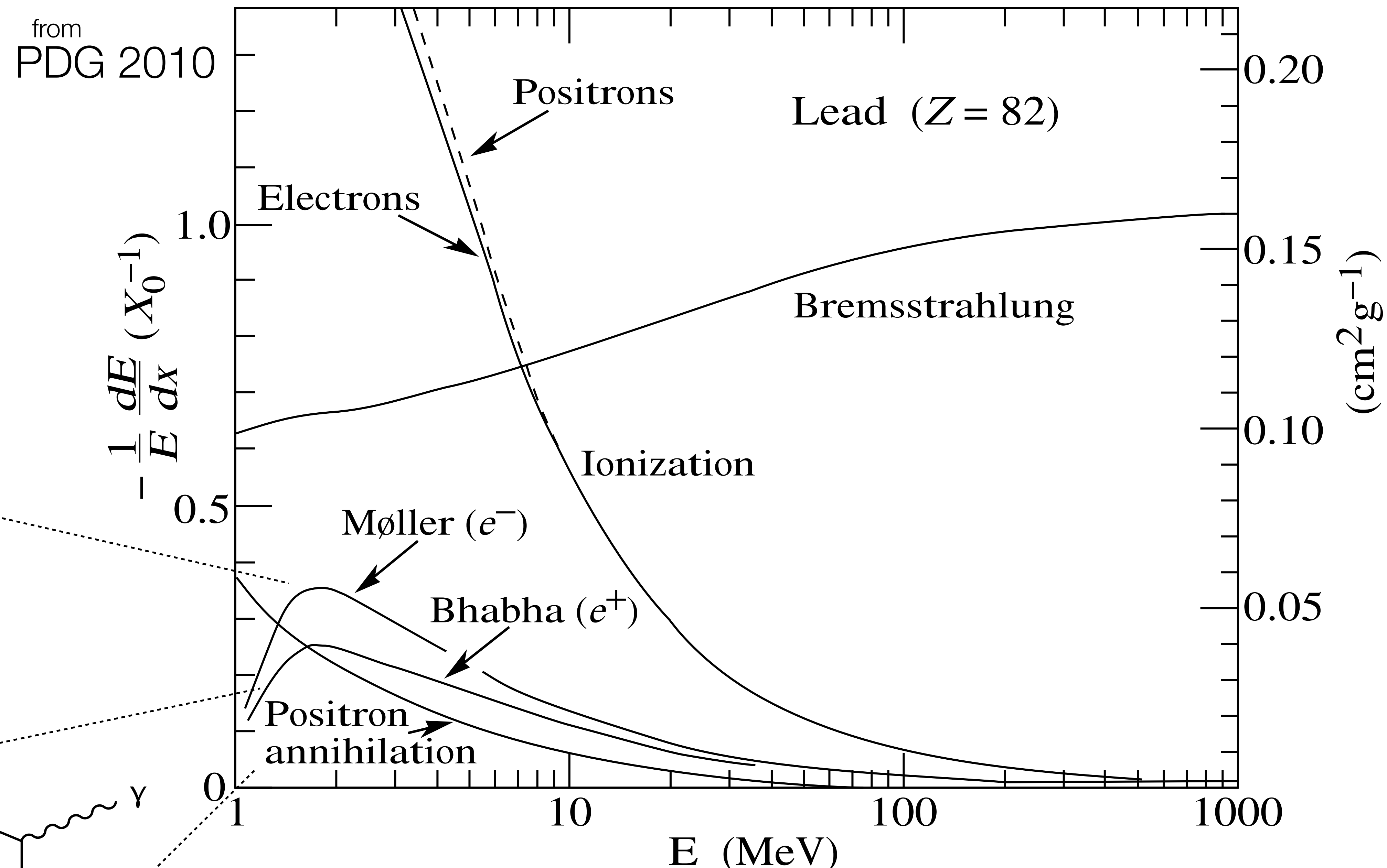
$k = E_\gamma$

Example Pb:

E_e = 10 GeV: Suppression for k < 23 MeV [y = 0.0023]
E_e = 100 GeV: Suppression for k < 2.3 GeV [y = 0.023]

Total Energy Loss of Electrons





Fractional energy loss per radiation length in lead as a function of electron or positron energy

Energy Loss – Summary Plot for Muons