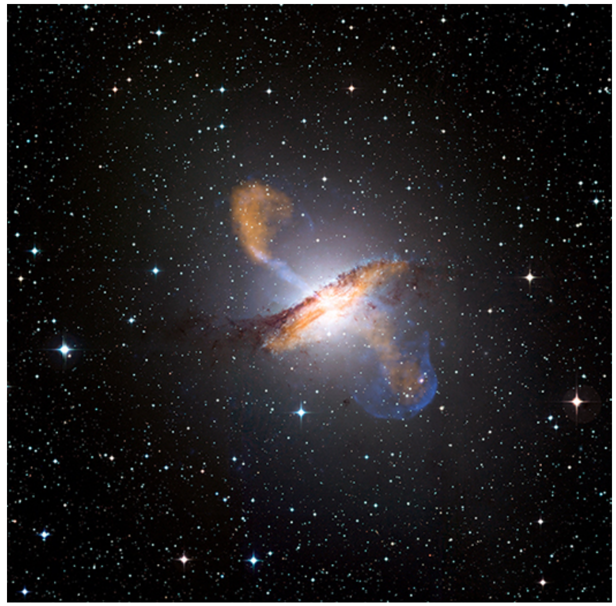
HIGH ENERGY ASTROPHYSICS - Lecture 9



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Wednesday

Hadronic Processes

1 Overview

- Reminder: Leptonic VHE origin - Inverse Compton
- Hadronic interactions with matter $(p + p)$ and radiation $(p + \gamma)$
- Cross-sections and inelasticities
- Reaction energy thresholds
- GZK cut-off etc.
- Resultant particle spectra
- Energy loss timescale

2 Leptonic Processes: Inverse Compton (IC) – Summary

- Total radiated IC power for single electron (Thomson regime, $\epsilon_i \ll m_e c^2/\gamma$):

$$P_{IC} = \frac{4}{3}\gamma^2\beta^2\sigma_T c U_{rad}, \quad \text{with} \quad U_{\rm rad} = \int n_{\rm ph}(\epsilon)\epsilon \, d\epsilon$$

  This is similar to single particle synchrotron power:

$$P_{syn} = \frac{4}{3}\gamma^2\beta^2\sigma_T c u_B, \quad \text{with} \quad U_B = \frac{B^2}{8\pi}$$

  Note: ICS (Thomson) for protons comes with reduced cross-section:

$$\sigma = \sigma_T \left(\frac{m_e}{m}\right)^2$$

- For relativistic electrons, head-on approximation (scattered photon follows e^--direction in lab frame) due to relativistic beaming useful.

- Average increase of photon energy in ICS (Thomson):

$$\nu_f = \frac{4}{3}\gamma^2\nu_i$$

- Spectral index of Thomson-scattered photon spectrum, $P_{IC}(\nu_f) \propto \nu_f^{-\alpha}$ off power-law electron distribution with index p, $n_e(\gamma) \propto \gamma^{-p}$, is

$$\alpha = \frac{p-1}{2}$$

This is similar to synchrotron as well.

- Compton up-scattering of synchrotron photons by the electrons themselves (**=SSC**) preserves the spectral index in the Thomson regime

$$\alpha_{IC} \simeq \alpha_{syn}$$

3 Hadronic Processes I

Inelastic collisions of energetic protons with ambient matter, e.g.,

$$\begin{aligned} p + p &\rightarrow p + p + \pi^0 \\ p + p &\rightarrow p + n + \pi^+ \\ p + p &\rightarrow p + p + \pi^+ + \pi^- ... \end{aligned}$$

Pions decay very quickly via main channels (probability $\geq 98.8\%$):

$$\begin{aligned} \pi^0 &\rightarrow \gamma + \gamma \\ \pi^+ &\rightarrow \mu^+ + \nu_\mu \quad \text{and} \quad \mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e \\ \pi^- &\rightarrow \mu^- + \bar{\nu}_\mu \quad \text{and} \quad \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e \end{aligned}$$

$\Rightarrow$ Neutrinos as *smoking gun* of hadronic processes.

- At high energies, 3 types of pions are produced with similar probabilities.
- Neutrinos produced with flavors (ratio):

$$\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$$

- After propagation, oscillations redistribute flavors

$$\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$$

4 Cross-section σ_{pp}

Cross-sections for single pion-production in inelastic pp-collisions:

- Total cross-section σ_{pp} grows quickly to several tens of mb for proton projectile $E_{kin} \sim$ few GeV, and then rises only weakly with energy.
(*Note:* mb=millibarn=10^{-27} cm^2 $\sim \sigma_T/500$).

- Below particle production threshold (E_{kin} = 280 MeV), cross-section becomes zero.

- Parameterization of experimental data (cf. Norbury 2009), e.g.,

$$\sigma_{pp\to\pi^0 X}(E_p) = \left(0.007 + 0.1\frac{\ln E_{kin}}{E_{kin}} + \frac{0.3}{E_{kin}^2}\right)^{-1} \quad \text{mb}$$

with $E_{kin} = E_p - m_p^2 c^2$ ($m_p c^2 = 0.94$ GeV) kinetic energy of projectile in lab frame **in units of GeV**.

- Inelasticity for proton energies in GeV-TeV range $\kappa_{\pi^0} \simeq 0.17$.

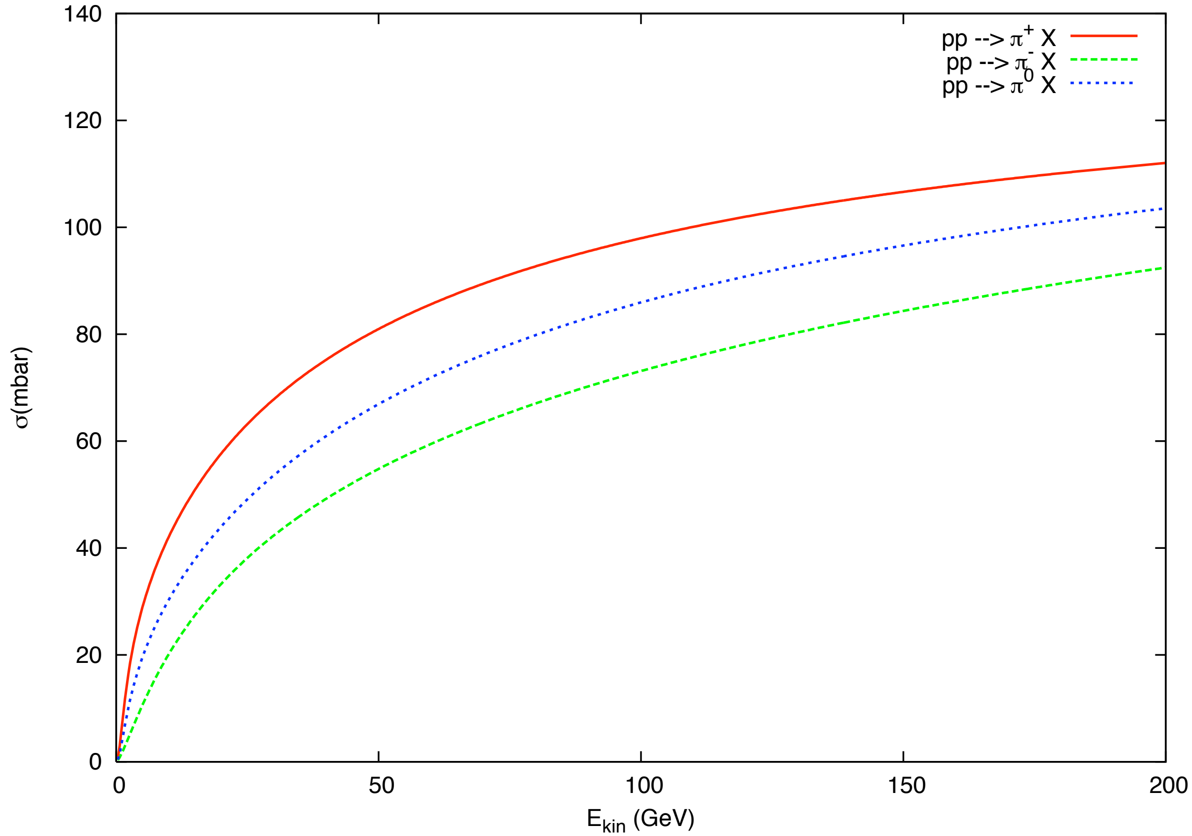

Figure 1: Parameterizations of total inclusive cross-sections for π-production in pp-collisions (following Norbury 2009). The cross-sections (probabilities) are similar, rising quasi-logarithmically towards high energies. It becomes zero for energies below 280 MeV. For comparison, note that the Thomson cross-section $\sigma_T = 665$ mbar.

5 Hadronic Processes II (*photo-meson-production*)

Interactions with radiation fields – dominant reactions:

$$\begin{aligned} p + \gamma &\rightarrow n + \pi^+ \\ p + \gamma &\rightarrow p + \pi^0 \end{aligned}$$

At high energies, multi-π-production dominates, e.g. $p + \gamma \rightarrow p + a\pi^0 + b(\pi^+ + \pi^-)$.

- Pion decay channels as before $\Rightarrow \gamma$, ν and $e^\pm$, e.g.

$$\begin{aligned} \pi^0 &\rightarrow \gamma + \gamma \\ \pi^+ &\rightarrow \mu^+ + \nu_\mu \quad \text{and} \quad \mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e \end{aligned}$$

- Cross-section (above threshold $E'_\gamma \simeq 145$ MeV), simplified parameterization:

$$\begin{aligned} \sigma_{p\gamma}(E'_\gamma) &\simeq 340\mu\text{barn} \qquad 200 \text{ MeV} \leq E'_\gamma \leq 500 \text{ MeV} \\ &\simeq 120\mu\text{barn} \qquad E'_\gamma \geq 500 \text{ MeV} \end{aligned}$$

  with E'_γ *measured in proton rest frame*, inelasticity $\kappa_{p\gamma} \simeq 0.2$ at low E'_γ energies $\Rightarrow$ mean energy of secondaries $E_\gamma \sim 0.1 E_p$ and $E_{e,\nu} \simeq 0.05 E_p$.

- Mean lifetimes: $\tau_{\pi^\pm} = 2.5 \times 10^{-8}$ s, $\tau_{\pi^0} = 8.4 \times 10^{-17}$ s.

- Masses: $m_{\pi^\pm} c^2 = 139.6$ MeV, $m_{\pi^0} c^2 = 134.97$ MeV

6 Center-of-Momentum (CoM) Frame and Energy Thresholds

CoM frame (denoted with $*$) for two particles m_1, m_2 defined by:

$$\boxed{\vec{p}_1^{\,*} + \vec{p}_2^{\,*} = 0}$$

In CoM frame many dynamical processes are symmetrical and easier to describe.

With $e := E/c = \gamma m_i c$ and $P = (e, \vec{p})$, total CoM 4-momentum becomes:

$$P^* = P_1^* + P_2^* = (e_1^* + e_2^*, 0)$$

Invariance of 4-vectors relates CoM and lab. frame quantities:

$$P^{*\mu} P^*_\mu = (P_1^* + P_2^*)^2 = P^\mu P_\mu = (P_1 + P_2)^2 =: M^2 c^2$$

with M=*invariant mass* (Mandelstam variable $s := M^2c^2$). CoM frame behaves like particle with rest mass M and energy momentum 4-vector $P = P_1 + P_2$.

CoM frame could be moving with Lorentz factor found from:

$$\gamma_{CM} = \frac{E}{Mc^2} = \frac{e_1 + e_2}{\sqrt{(P_1 + P_2)^2}} = \frac{e_1 + e_2}{\sqrt{(e_1 + e_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}}$$

For collision of energetic proton with proton at rest $(e_2 = m_p c, \vec{p}_2 = 0)$:

$$\gamma_{CM} = \frac{e_1 + e_2}{\sqrt{(e_1 + e_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}}$$

$$= \frac{\gamma_p m_p c + m_p c}{\sqrt{(\gamma_p m_p c + m_p c)^2 - (\gamma_p m_p v)^2}} = \frac{\gamma_p + 1}{\sqrt{(\gamma_p + 1)^2 - \gamma_p^2 \frac{v^2}{c^2}}} = \frac{\gamma_p + 1}{\sqrt{(\gamma_p + 1)^2 - (\gamma_p^2 - 1)}}$$

$$= \frac{\gamma_p + 1}{\sqrt{2\gamma_p + 2}} = \sqrt{\frac{\gamma_p + 1}{2}}$$

Incidentally, this is also Lorentz factor of the protons in CoM frame.

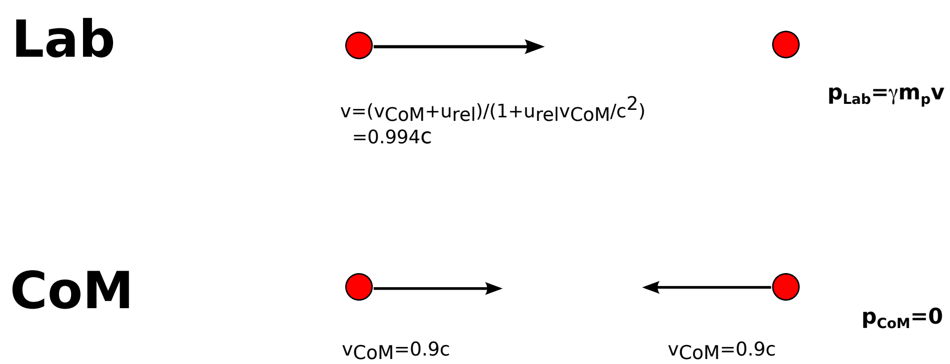

Figure 2: The same collision as viewed in the lab and in the CoM frame.

Consider creation of particle with mass δm **in CoM frame** of two particles 1 and 2:

$\Rightarrow$ Total energy must be at least **threshold energy**:

$$E^*_{th} = M^*_{th} c^2 = m_1 c^2 + m_2 c^2 + \delta m c^2$$

At same time **in lab. frame** (due to invariance):

$$\begin{aligned} M^{*2} c^2 &= P^2 = (e_1 + e_2, \vec{p}_1 + \vec{p}_2)^2 = (e_1 + e_2, \vec{p}_1 + \vec{p}_2)(e_1 + e_2, -[\vec{p}_1 + \vec{p}_2]) \\ &= (e_1^2 - \vec{p}_1^{\,2}) + (e_2^2 - \vec{p}_2^{\,2}) + 2e_1 e_2 - 2\vec{p}_1 \vec{p}_2 = m_1^2 c^2 + m_2^2 c^2 + 2e_1 e_2 - 2\vec{p}_1 \vec{p}_2 \end{aligned}$$

Demanding $M^*_{th} c^2 \leq M^* c^2$ to produce particle thus implies:

$$M_{th}{}^{*2}\, c^2 = \frac{1}{c^2}(m_1 c^2 + m_2 c^2 + \delta m c^2)^2 \stackrel{!}{=} m_1^2 c^2 + m_2^2 c^2 + 2e_1 e_2 - 2p_1 p_2 \cos\theta$$

with θ angle between momentum vectors of the colliding particles, or

$$(m_1^2 c^2 + m_2^2 c^2 + [\delta m]^2 c^2 + 2m_1 m_2 c^2 + 2m_1 \delta m\, c^2 + 2m_2 \delta m\, c^2) = m_1^2 c^2 + m_2^2 c^2 + 2e_1 e_2 - 2p_1 p_2 \cos\theta$$

$$\boxed{e_1 e_2 - p_1 p_2 \cos\theta = m_1 m_2 c^2 + \delta m\ c^2\ (m_1 + m_2 + 0.5\ \delta m)}$$

Remember:

$$\boxed{e_1 e_2 - p_1 p_2 \cos\theta = m_1 m_2 c^2 + \delta m\ c^2\ (m_1 + m_2 + 0.5\ \delta m)}$$

- If both particles have rest mass (no $photons$; $p_i = \sqrt{\gamma_i^2 - 1}\, m_i c$, $e_i = \gamma_i m_i c$), then this results in

$$\gamma_1 \gamma_2 - \sqrt{\gamma_1^2 - 1}\sqrt{\gamma_2^2 - 1}\ \cos\theta = 1 + \delta m \left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{\delta m}{2 m_1 m_2} \right)$$

Example: Consider reaction $p + p \rightarrow p + p + \pi^0$, with one proton initially at rest ($\vec{p}_2 = 0$, $\gamma_2 = 1$), e.g., collision of cosmic ray particle with hydrogen nucleus in ISM, $\delta m = m_{\pi^0}$:

$$\gamma_1 = 1 + m_{\pi^0} \left(\frac{2}{m_p} + \frac{m_{\pi^0}}{2 m_p^2} \right) \simeq 1.2984$$

for $m_{\pi^0} = 134.97$ MeV/c^2 and $m_p = 938.27$ MeV/c^2.

$\Rightarrow$ in lab. frame $E_{p,kin} = (\gamma_1 - 1) m_p c^2 \simeq 280$ MeV needed to produce pion, although its rest mass is only ~ 135 MeV (as particles also have momentum after collision).

Remember:

$$\boxed{e_1 e_2 - p_1 p_2 \cos\theta = m_1 m_2 c^2 + \delta m\ c^2\ (m_1 + m_2 + 0.5\ \delta m)}$$

- If one particle is a photon ($e_2 = p_2$ and $m_2 = 0$), then threshold energy

$$e_2\ \left(\gamma_1 - \sqrt{\gamma_1^2 - 1}\ \cos\theta\right) = \delta m\ c\ \left(1 + \frac{\delta m}{2m_1}\right)$$

Example: Consider reaction $p+\gamma \to p+\pi^0$ on CMB photons (mean energy $E_2 =< h\nu >\simeq 3kT \simeq 7 \times 10^{-4}$ eV [SI], $e_2 = E_2/c$). Threshold energy for most favourable collision angle ($\cos\theta = -1$ head-on) for high γ_1:

$$\Rightarrow 2\gamma_1 \simeq \frac{m_{\pi^0} c}{e_2} \left(1 + \frac{m_{\pi^0}}{2m_p}\right) \quad \text{or} \quad \gamma_1 \simeq 10^{11}$$

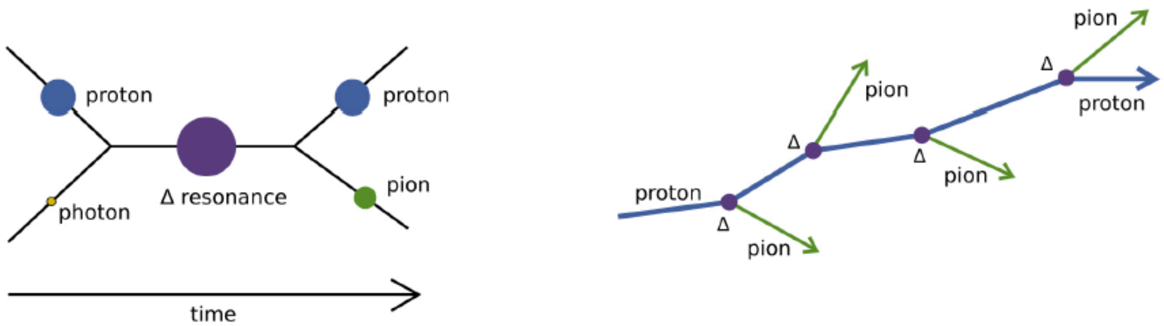

Figure 3: Left: Photo-pion production via excitation of a Δ-resonance in collision of an ultra-high energy cosmic ray proton with a CMB photon. Right: Trajectory of a super-GZK proton through the CMB, suffering attenuation due to repetitive photo-pion production [Credits: W. Bietenholz, arXiv:1305.1346].

⇒ Greisen-Zatsepin-Kuzmin (GZK) cutoff

⇒ Expect suppression of UHECR flux above $E_{CR} = \gamma_1 m_p c^2 \sim 10^{20}$ eV

Associated cooling timescale:

$$t_c(E) = \frac{E}{dE/dt} \simeq \frac{E}{\frac{\Delta E}{\Delta\tau}} = \frac{E}{\frac{\Delta E}{\Lambda/c}} = \frac{E}{\kappa_{p\gamma} E}\frac{\Lambda}{c} = \frac{1}{n_\gamma \; \sigma_{p\gamma} \; c \; \kappa_{p\gamma}}$$

with mean free path $\Lambda = c\,\Delta\tau = \frac{1}{n\sigma}$

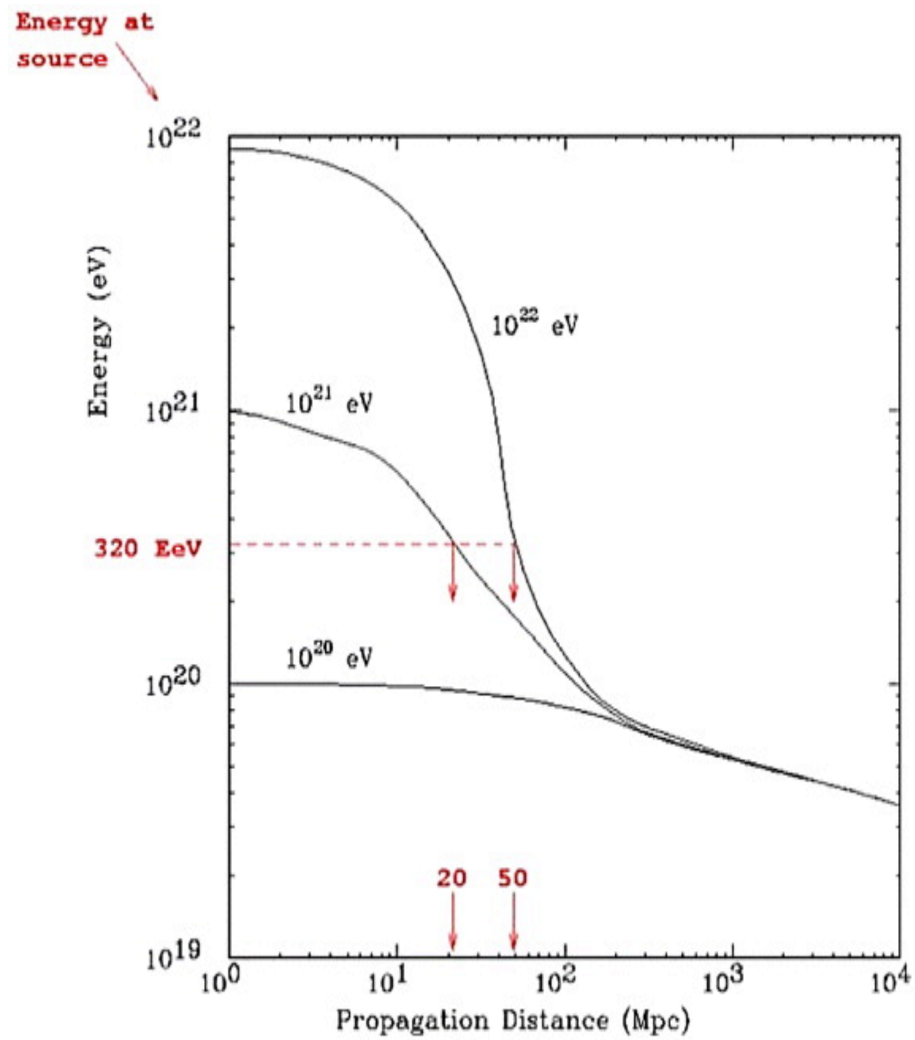

Figure 4: The energy of protons as a function of the propagation distance. As a consequence of the GZK, protons with energy above the threshold for photo-pion production lose energy as they travel in space (we can associate a cooling time scale $t_c(E) \sim 1/(n_{\gamma,E}\sigma c \kappa_{p\gamma})$ with it. If we see ultra-high energy cosmic ray protons (with 3×10^{20} eV), then they must come from sources at $d \lesssim 100$ Mpc [Credits: N. Matthiae, New J. Phys 12 (2010)].

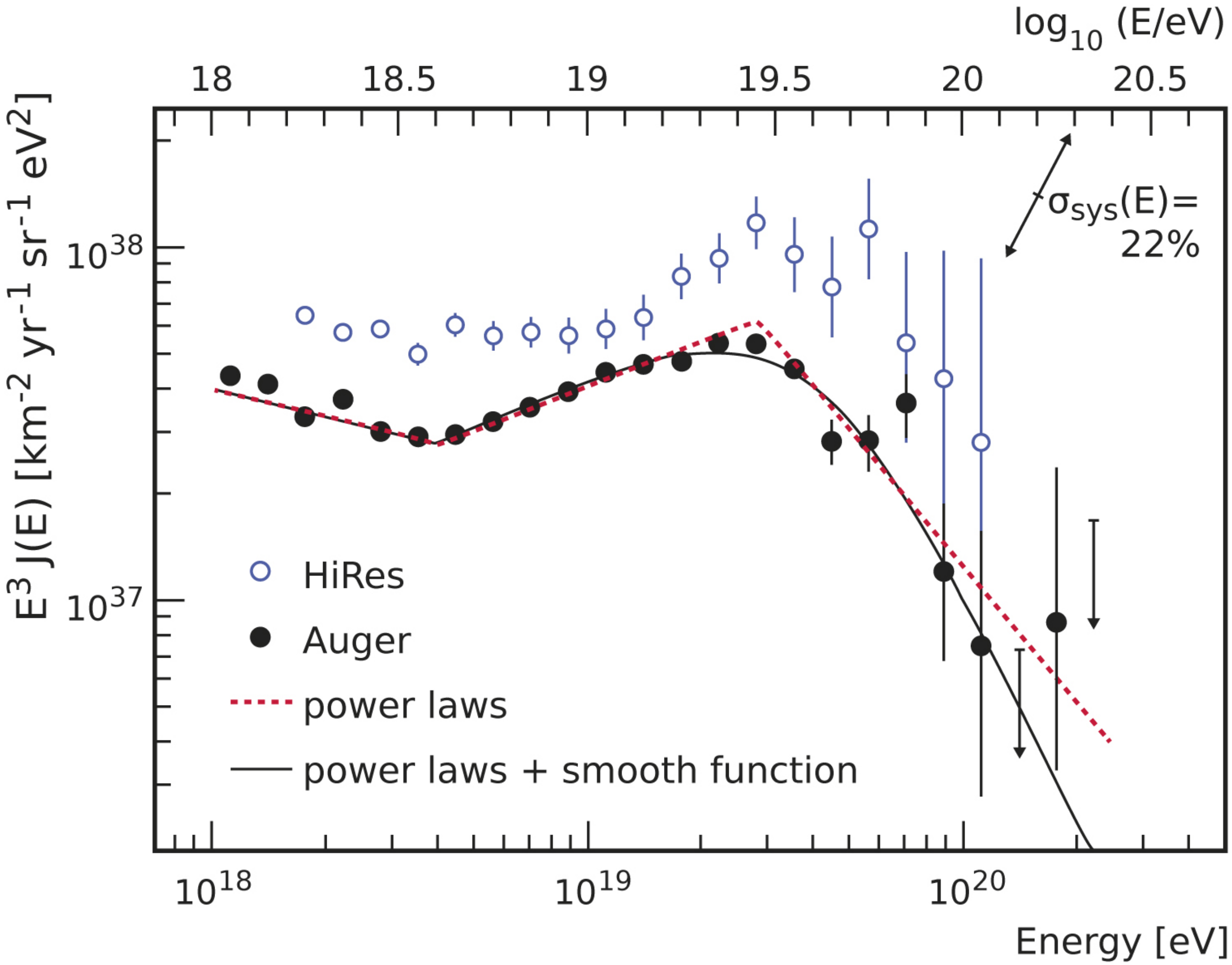

Figure 5: Energy spectrum of Cosmic Rays (multiplied by E^3) above 10^{18} eV as measured by the HiRes and Auger Experiments. Above $\sim 3 \times 10^{19}$ a sharp decline in flux is seen, which could either be related to GZK suppression or a limited particle acceleration efficiency in the sources [Credits: PA Collaboration, Phys. Lett. B 685 (2010)].

Remember:

$$\boxed{e_1 e_2 - p_1 p_2 \cos\theta = m_1 m_2 c^2 + \delta m\ c^2\ (m_1 + m_2 + 0.5\ \delta m)}$$

- If both particles are photons ($e_i = p_i$ and $m_i = 0$), e.g., as for pair creation, energy threshold (head-on collision, $\cos\theta = -1$) becomes:

$$e_1 e_2 = 0.25\ \delta m^2\ c^2$$

7 Resulting pion (π) spectrum in pp-interactions:

- Delta-Function Approximation: Can approximate mean pion kinetic energy E_π to that of proton E_{kin} assuming

$$E_\pi = \kappa_\pi E_{kin} \quad \text{with} \quad \kappa_{\pi^0} \sim 0.17 \quad (\text{"inelasticity" for } \pi^0)$$

- Pion emissivity q_π=*number of pions produced per unit volume, time and energy* for incident proton energy distribution $N_p(E_p)$ [1/erg cm^3] and density of target nuclei n_H, i.e. $Q = q\,V = N/\Delta\tau$:

$$q_\pi(E_\pi) \simeq c n_H \int \delta(E_\pi - \kappa_\pi E_{kin})\ \sigma_{pp}(E_p)\ N_p(E_p) dE_p$$

- Can simplify with $E_{kin} = E_p - m_p c^2$ for proton kinetic energy, and using substitution $\tilde{E}_p = \kappa_\pi E_p$:

$$\begin{aligned} q_\pi(E_\pi) &\simeq c n_H \int \delta(E_\pi + \kappa_\pi m_p c^2 - \tilde{E}_p)\ \sigma_{pp}\left(\frac{\tilde{E}_p}{\kappa_\pi}\right) N_p\left(\frac{\tilde{E}_p}{\kappa_\pi}\right) \frac{d\tilde{E}_p}{\kappa_\pi} \\ &\simeq \frac{c n_H}{\kappa_\pi}\ \sigma_{pp}\left(m_p c^2 + \frac{E_\pi}{\kappa_\pi}\right)\ N_p\left(m_p c^2 + \frac{E_\pi}{\kappa_\pi}\right) \ \propto\ \sigma_{pp}(E_p)\ N_p(E_p) \end{aligned}$$

$\Rightarrow$ Shape of the pion spectrum is similar to shape of the proton spectrum but shifted to lower energy by κ_π.

8 Resulting γ-spectrum from π^0-decay ($\pi^0 \to 2\gamma$):

- In pion center of mass reference (CoM) frame, mean photon energy $E^*_\gamma = \frac{1}{2} m_{\pi^0} c^2 \simeq 67.5$ MeV.
- In lab. frame (Doppler shift):

$$E_\gamma = D\, E^*_\gamma = \gamma_\pi \frac{m_{\pi^0} c^2}{2} (1 + \beta_\pi \cos\theta^*)$$

with θ^* measured in CoM frame (!).

- γ-rays isotropically distributed in CoM = pion rest frame:
$\Rightarrow$ Number of γ-rays emitted into angles θ^* and $\theta^* + d\theta^*$ (cf. solid angle):

$$n_\gamma(\theta^*) d\theta^* \propto \sin\theta^* d\theta^* = -d\cos\theta^*$$

$\Rightarrow$ Number of γ-rays with lab-frame energy between E_γ and $E_\gamma + dE_\gamma$:

$$n_\gamma(E_\gamma) = n_\gamma(\theta^*) \frac{d\theta^*}{dE_\gamma} \propto \frac{d\cos\theta^*}{dE_\gamma}$$

- From Doppler (see above):

$$dE_\gamma = \gamma_\pi \frac{m_{\pi^0} c^2}{2} \beta_\pi \; d\cos\theta^* = \frac{m_{\pi^0} c^2}{2} \sqrt{\gamma_\pi^2 - 1} \; d\cos\theta^*$$

So:

$$\frac{d\cos\theta^*}{dE_\gamma} = \frac{1}{\frac{m_{\pi^0}c^2}{2}\sqrt{\gamma_\pi^2 - 1}} = \frac{2}{\sqrt{E_\pi^2 - m_{\pi^0}^2 c^4}}$$

$\Rightarrow$ For mono-energetic distribution of neutral pions, photon spectrum is a constant:

$$\begin{aligned} P(E_\gamma|E_\pi) &= \frac{2}{\sqrt{E_\pi^2 - m_{\pi^0}^2 c^4}} \quad \text{if} \quad \frac{E_\pi}{2}(1-\beta_\pi) \leq E_\gamma \leq \frac{E_\pi}{2}(1+\beta_\pi) \\ &= 0 \quad \text{else} \end{aligned}$$

- since $\frac{1}{2}(\log E_{\gamma,max} + \log E_{\gamma,min}) = \log(E_{\gamma,max}E_{\gamma,min})^{1/2} = \log(E_\pi^2[1-\beta_\pi^2]/4)^{1/2} = \log\frac{m_{\pi^0}c^2}{2}$, in log-representation the centre of the interval is half the pion rest mass (holds for every E_π).

$\Rightarrow$ Resulting γ-spectrum always has broad bump centred at 67.5 MeV independently of the shape of the pion-distribution and thus of the parent proton distribution.

- Note: $E_{\gamma,min} + E_{\gamma,max} = E_\pi$ and $E_{\gamma,min} E_{\gamma,max} = \frac{E_\pi^2}{4}(1-\beta_\pi^2) = \frac{m_{\pi^0}^2 c^4}{4}$. Thus, minimum π^0-energy to produce photon of energy $E_{\gamma,min} = E_\gamma$ is

$$E_{\pi,min} = E_\gamma + E_{\gamma,max} = E_\gamma + \frac{m_{\pi^0}^2 c^4}{4E_\gamma}$$

Alternatively: $E_{\gamma,min} = \frac{m_\pi c^2}{2}\gamma_\pi(1-\beta_\pi) = \frac{m_\pi c^2}{2}\sqrt{\frac{(1-\beta_\pi)}{(1+\beta_\pi)}}$. Solving for β_π gives $\beta_\pi = \frac{(1-a^2)}{(1+a^2)}$, where $a := 2E_{\gamma,min}/(m_\pi c^2)$. So $E_{\pi,min} = \gamma_\pi m_\pi c^2 = \frac{1+a^2}{2a} m_\pi c^2 = E_\gamma + \frac{m_{\pi^0}^2 c^4}{4E_\gamma}$ is minimum pion energy to produce photon with energy E_γ.

- Thus γ-ray emissivity $q_\gamma(E_\gamma)$ [erg^{-1} cm^{-3} s^{-1}] becomes

$$\begin{aligned} q_\gamma(E_\gamma) &= \int_{E_{\pi,min}}^{\infty} P(E_\gamma|E_\pi)\, q_\pi(E_\pi) dE_\pi \\ &= 2\int_{E_\gamma + m_{\pi^0}^2 c^4/4E_\gamma}^{\infty} \frac{q_\pi(E_\pi)}{\sqrt{E_\pi^2 - m_{\pi^0}^2 c^4}} dE_\pi \end{aligned} \tag{1}$$

- At high energy (well above bump), γ-ray spectrum has spectral index similar to that of protons, but shifted in energy by factor κ_{π^0}. Mean energy of the photons is of order of $\frac{\kappa_{\pi^0}}{2} E_p \sim 0.1 E_p$.

9 Spectrum of ν produced in $\pi^+ \to \mu^+ \nu_\mu$

- In CoM frame we have energy-momentum conservation:

$$P_\pi^* = P_\mu^* + P_\nu^*$$

with 4-vectors $P_\pi^* = (m_\pi c, 0)$, $P_\mu^* = (E_\mu^*/c, \vec{p}_\mu^*)$, and $P_\nu = (E_\nu^*/c, \vec{p}_\nu^*)$ (massless):

$$(P_\pi{}^* - P_\nu{}^*)^2 = (P_\mu{}^*)^2$$

$$\Rightarrow P_\pi^*{}^2 + P_\nu^*{}^2 - 2P_\pi^* P_\nu^* = m_\mu^2 c^2$$

$$\Rightarrow m_\pi^2 c^2 + 0 - 2m_\pi E_\nu^* = m_\mu^2 c^2$$

Thus

$$\boxed{E_\nu^* = \frac{m_\pi^2 c^4 - m_\mu^2 c^4}{2m_\pi c^2} \simeq 29.8 \text{ MeV}}$$

for $m_\mu = 105.6$ MeV/c^2 and $m_{\pi^0} = 135$ MeV/c^2.

- *Note:* Neutrino is assumed to be massless, so $\vec{p}_\nu{}^* = E_\nu^*/c$. In CoM frame, $|\vec{p}_\mu{}^*| = |-\vec{p}_\nu{}^*| = E_\nu^*/c$, so $E_\mu^* = \sqrt{\vec{p}_\mu{}^{*2} c^2 + m_\mu^2 c^4} \simeq 110$ MeV, i.e. kinetic energy $E_{\mu,kin}^* = E_\mu^* - m_\mu c^2 = 4$ MeV

- In observer frame, assuming $\gamma_\pi \gg 1$, we have (cf. Doppler shift)

$$0 \leq E_\nu \leq E_{\nu,max} = \gamma_\pi E_\nu^*(1+\beta_\pi) \simeq 2\gamma_\pi \frac{m_\pi^2 c^4 - m_\mu^2 c^4}{2m_\pi c^2}$$
$$= \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) E_\pi =: \lambda E_\pi \simeq 0.427 E_\pi$$

- Neutrino emissivity can be written as (with $\lambda := 1 - m_\mu^2/m_\pi^2$):

$$q_\nu(E_\nu) \simeq \int_{E_\nu/\lambda}^{\infty} \frac{q_\pi(E_\pi)}{\lambda E_\pi} dE_\pi$$

Using, similarly as before, $E_\nu = \gamma_\pi E_\nu^*(1+\beta_\pi \cos\theta^*)$, i.e. $dE_\nu = \gamma_\pi E_\nu^* \beta_\pi \; d\cos\theta^*$, so that

$$\frac{d\cos\theta^*}{dE_\nu} = \frac{1}{\gamma_\pi E_\nu^* \beta_\pi} = \frac{1}{\sqrt{\gamma_\pi^2 - 1}\; E_\nu^*} = \frac{1}{\sqrt{\gamma_\pi^2 - 1}\frac{m_\pi c^2}{2}\lambda} \simeq \frac{2}{\lambda E_\pi}$$

noting that for the 2-body decay, $P(E_\nu|E_\pi) = \frac{1}{2}\; \frac{d\cos\theta^*}{dE_\nu}$

- Neutrino spectrum has spectral index close to that of protons.

10 Proton Energy Losses

- What is typical time scale for γ-ray production via pp (variability)?
- Assume $\sigma_{pp} \sim 40$ mb and ambient density n_H [cm^{-3}].
- Fraction $\kappa_{\pi^0} \simeq 0.17$ of proton kinetic energy is transferred to π^0
  $\Rightarrow$ power radiated in γ-rays for a single proton:

$$\frac{dE_r}{dt} \simeq \frac{\Delta E}{\Delta t} = \frac{\kappa_\pi^0 E}{\frac{1}{n_H \sigma_{pp} c}} = \kappa_{\pi^0} E n_H \sigma_{pp} c$$

- Gamma-ray luminosity for proton distribution $n_p(E)$:

$$L_\gamma = \int \frac{dE_r}{dt} n_p(E) dE = \kappa_{\pi^0} n_H \sigma_{pp} c \int n_p(E) E dE =: \frac{W_p}{t_\gamma}$$

  where W_p is total energy in energetic proton population.

- Timescale t_γ for gamma-ray production is large:

$$t_\gamma = \frac{1}{\kappa_{\pi^0} n_H \sigma_{pp} c} \simeq 5 \times 10^{15} \, (1 \text{ cm}^{-3}/n_H) \quad \text{sec}$$

  This is usually large $\Rightarrow$ variability can disfavour pp-interactions